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Assessment of the Functional Capabilities of Free-Flying Dynamically Similar Aircraft Models

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Abstract:

This paper discusses two special approaches developed by the author to assess the similarity and functional capabilities of a free-flying, dynamically similar model (FDSM) in the design of aircraft flight dynamics modelling. When implementing the first approach, the motion of the FDSM is constantly influenced by the control laws of the FDSM Automatic Control System (ACS), similar to those of a real aircraft, or it simulates the pilot's behavior of the aircraft itself. The second approach assumes the failure of aviation equipment (including ACS), the loss of flight crew capability, and the aircraft entering critical or dangerous uncontrolled flight modes. The results of calculations for FDSM in real design are considered as an approbation of the obtained determining conditions.

Keywords:

assessment of the design, functional capabilities, free-flying dynamically similar aircraft models

1 Introduction

The method of studying the flight characteristics of aircraft on their free flying dynamically similar models (FDSM) does not exclude, but complements, other experimental aerodynamics methods. If it is necessary to carry out extensive parametric studies of aerodynamic characteristics under steady flow conditions, wind tunnel (WT) testing remains the most effective method. When it is necessary to study critical flight regimes associated with aggressive maneuvers and complex spatial motion, flight testing on an FDSM is practically the only method for obtaining reliable information at the design stage of a full-scale aircraft [1].

Of course, the most reliable data on flight characteristics can be obtained from testing full-scale aircraft, but only at the final stage of the entire set of works. Making

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changes to the design of the aircraft under development based on the test results of its prototype requires extremely high costs and leads to delays in the commissioning of serial aircraft [1].

The methodological gap between aerodynamic testing of models in WT and flight testing of full-scale aircraft was bridged by the development of the method for studying flight characteristics on FDSM – autonomous unmanned experimental reusable aerial vehicles capable of performing remotely piloted or automatic flights according to a specified program. In other words, an FDSM is an unmanned research aircraft (Fig. 1).

Fig. 1 FDSM of a prospective aircraft

In several countries, the development of the theoretical and methodological framework for the design, manufacturing, ground and flight testing of FDSM, as well as the conduct of flight research using them, has been carried out. Examples of high efficiency can be found in the programs carried out in the United States, which involved the construction of FDSM and advanced flight dynamics research for the military aircraft McDonnell Douglas F-4 and F-15, Northrop F-18 and F-5, Grumman F-14 and X-29, General Dynamics YF-16, as well as the FFD model developed by Rockwell under the HiMAT program and the X-31. Similar examples, but for transport aircraft, include the program for developing the X-48 NASAN FDSM, as well as the program for evaluating new technologies on the FDSM of the experimental aircraft PTERA, based on the Boeing 737 passenger aircraft [2].

The development and implementation of a method for studying aircraft flight characteristics using free-flying dynamically similar models (FDSM) became possible through the work of specialists at the Research Institute of Flight Regimes Physical Modelling (RIPFM) of the National Aerospace University “Kharkiv Aviation Institute,” and its subsequent adoption within aviation industry enterprises in the USSR and Ukraine. The theoretical foundations of the method for investigating critical flight regimes of aircraft using FDSM, as developed by them, along with the methodological framework for their creation, are based on extensive experimental data from real projects [3, 4].

The conceptual design of an FDSM of aircraft is developed based on the analysis of its flight characteristics and research capabilities. At the same time, the rational values of the main parameters of the FDSM of aircraft are determined, and efforts are made to achieve equality between its available and required values, to meet the flight performance requirements (FPR) for the FDSM, to ensure a defined flight regime envelope of the model (FREM), and to provide the necessary envelope of aircraft flight regimes that can be simulated (EAFRS). The result of the calculation and analysis of the FREM is a conclusion on the altitude-speed design characteristics of the FDSM, but a conclusion regarding its potential as a research tool can be made only after constructing the EAFRS and comparing it with the flight regimes specified for investigation within the aircraft flight regime envelope (AFRE). It should be noted that when constructing the altitude–speed limitation system of the FREM, functional dependencies on the mass and geometric parameters of the FDSM of the modeled aircraft are used, but not on the moments of inertia (MoI) of the FDSM. an FDSM Accordingly, a definitive assessment of the flight characteristics and functional capabilities of an aircraft FDSM as a research tool requires further theoretical analysis and supporting calculations [5, 6].

At present, there are several different approaches to assessing the functional capabilities of aircraft, but they have their limitations and cannot be fully applied to FDSM of aircraft [7].

In this work, the author proposes two specific approaches for assessing the similarity and functional capabilities of FDSM in modeling the flight dynamics of aircraft [8].

In the first approach, the motion of the FDSM is considered to be continuously under the influence of the control laws of its automatic control system (ACS), like that of a full-scale aircraft, or it simulates the behavior of the aircraft's pilot. Such assumptions can be attributed to the stable and controllable motion of an aircraft (either the full-scale aircraft or its FDSM.

However, even in routine flight operations, cases of aircraft failure (including the ACS), loss of flight crew performance, and the aircraft entering critical or dangerous uncontrolled flight regimes are not uncommon. The study of these flight regimes, including entering and recovering from them, is precisely the prerogative of creating FDSM for flight research and forms the basis of the second approach.

The purpose of this study is to conduct research aimed at analyzing and evaluating the existing first approach, as well as developing the theoretical foundations of a new, second approach for assessing the design functional capabilities of an aircraft FDSM in the event of an ACS failure.

2 Literature Review on Approaches to Assessing Aircraft FDSM Functional Capabilities in Design

The assessment of the degree of similarity in both approaches is proposed to be carried out by comparing the behavior of the geometrically identical, actually designed FDSM of the aircraft (the "real" model) with that of the reference FDSM, which fully satisfies the similarity conditions [7].

From this standpoint, for the implementation of the first approach, the author substantiates the use of numerical simulation methods and the well-known mathematical framework from automatic control theory for evaluating ACS performance in the problem of selecting quality indicators (integral performance measures of the ACS) and determining the functional capabilities of aircraft FDSM in the design process. The theoretical foundations for determining the suitability of an aircraft FDSM as a research tool have been developed.

In the first approach, comparing the behavior of the real and benchmark FDSM in numerical modeling, the behavior of the real and reference FDSM is compared using quality indicators in the form of integral estimates [8].

$$\left.\begin{aligned} I_0 &= \int_0^{\infty} \left[x(t) - x_{\infty}\right]^2 \mathrm{d}t \\ I_1 &= \int_0^{\infty} \left\{\left[x(t) - x_{\infty}\right]^2 + \tau_1^2 \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2\right\} \mathrm{d}t \\ &\vdots \\ I_n &= \int_0^{\infty} \left\{\left[x(t) - x_{\infty}\right]^2 + \tau_1^2 \left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \tau_2^4 \left(\frac{\mathrm{d}^2 x}{\mathrm{d}t^2}\right)^2 + \ldots + \tau_1^{2n} \left(\frac{\mathrm{d}^n x}{\mathrm{d}t^n}\right)^2\right\} \mathrm{d}t \end{aligned}\right\} \tag{1}$$

where $\tau_1, \tau_2, \cdots, \tau_n$ – the constant quantities having the dimension of time (the methods of their determination are cumbersome, but do not cause difficulties when using computational techniques [9, 10]), [s]; t – the current time, [s]; x – the parameter value in a transient process; $x_{\infty} = \lim_{t \to \infty} x(t)$.

Integral quality estimates (1) are generalized indices that allow us to investigate the nature of the transient dynamic component of the error of the system $\varepsilon = x(t) - x_{\infty}$ (and, $\mathrm{d}\varepsilon/\mathrm{d}t = \mathrm{d}x/\mathrm{d}t$, since $x_{\infty} = \mathrm{const}$). If the values of integral quality assessments of ACS for the real and benchmark FDSM do not differ by more than 5 % (by analogy with the transient damping), we can say that the behavior of both FDSM under the same external influence differs insignificantly.

The conclusion about the possibility of obtaining reliable data on the flight characteristics of the aircraft on its partially similar FDSM is made on the basis of:

- numerical modelling, in which the calculated values of integral quality assessments of ACS of real and benchmark FDSM (obtained from the results of integration of equations of the mathematical model of motion of the FDSM under typical single impacts) are determined and compared,
- flight tests of an FDSM with determination of the actual type of m-dimensional transient process at typical single influences (as in the case of numerical simulation), calculation and comparison of integral quality assessments of the automatic control system of a real FDSM based on flight data and the benchmark model based on the calculated values of numerical simulation.

It is clear from the description of the works that the same mathematical apparatus for calculating and comparing quality assessments is used when carrying out numerical modelling and flight tests of an FDSM. The difference is only in the method of obtaining the m-dimensional transient parameters of a real FDSM.

A number of theoretical studies (e.g., [9, 10]) and software implementations (one of which is described in [9] and used in this paper) are devoted to the calculation of the parameters of the s-dimensional transient process for an aircraft with a control system. This frees us from the need to write down the known but cumbersome full equations of the mathematical model of aircraft motion and to describe the algorithm of their integration on a computer.

Note only that, when solving the problem under consideration, the input quantities of interest are typical single actions in the longitudinal (e.g., by a stepwise change in the angle $\delta_{\mathrm{el.m.}}$ of elevator deviation) and lateral (e.g., by a stepwise change in the angle $\delta_{\mathrm{rud.m.}}$ of rudder deviation) control channels of the FDSM. The initial state of an

FDSM is steady horizontal flight corresponding to the initial pair (V_m, H_m). The number of pairs (V_m, H_m) is determined on the basis of the type of the flight regime area of a full-scale aircraft that can be modelled on an FDSM and the necessary expediency of conducting research in specific limiting and intermediate points of the flight regime area of an FDSM [2].

The behavior of an FDSM is evaluated by the values of the parameters of motion of the center of mass of the FDSM (air velocity V_m, [m/s]; flight altitude H_m, [m]), angles (roll γ_m, [rad]; pitch ν_m, [rad]; yaw $\emptyset_m$, [rad]; attack α_m, [rad]; glide β_m, [rad]), angular velocities (relative to the OX axis – ω_{xm}, [rad/s]; relative to the OY axis – ω_{ym}, [rad/s]; relative to the OZ axis – ω_{zm}, [rad/s]), overloads (along the OX axis – n_{xm} , along the OY axis – n_{ym}, along the OZ axis – n_{zm}). Thus, for a free-flying dynamically similar model, the parameter $s = 13$ (although this value can be increased if desired).

For a particular single action and the values of one of the initial pairs (V_m, H_m) according to the formulas (1) find $k \times s$ ($k = n + 1$) integral quality assessments, which can be written by the matrix of integral quality assessments of the ACS of an FDSM in the following form:

$$\left[I_n\right]=\begin{bmatrix} I_{11} & I_{12} & \cdots & I_{is} \\ I_{21} & I_{22} & & I_{2s} \\ \vdots & & \ddots & \vdots \\ I_{k1} & I_{k2} & \cdots & I_{ks} \end{bmatrix}=\left[I_{ij}\right] \tag{2}$$

When considering p single actions in r points of the investigated region of flight modes, on the basis of the matrices of integral evaluations (2) for the value of a particular type of single action and the value of one of the initial pairs (V_m, H_m), it is possible to obtain a general matrix [I_n^{gen}] of integral evaluations of the quality of the ACS of a FDSM. The size of the matrix [I_n^{gen}] – $k \times s \times p \times r$, and its element – $\underline{I_i}_{\,\mathrm{juv}}$. It is inconvenient to operate with matrices [I_{nr}^{gen}] and [I_{nb}^{gen}] of the real and benchmark FDSM, since its elements have different dimensionality. The only way is to perform joint matrix transformations so that the elements of the new matrix are dimensionless. In other words, the elements of the new matrix must form a dimensionless metric space [2, 10].

Such a matrix can be a matrix [$\delta I_n^{\mathrm{gen}}$], composed of elements that are relative errors of integral estimates. The value of each element of the matrix [$\delta I_n^{\mathrm{gen}}$] is found by the following formula:

$$\delta I_{i\,\mathrm{juv}} = \frac{I_{i\,\mathrm{juv}}^{\mathrm{b}} - I_{i\,\mathrm{juv}}^{\mathrm{r}}}{I_{i\,\mathrm{juv}}^{\mathrm{b}}} \tag{3}$$

where $I_{i\,\mathrm{juv}}^{\mathrm{b}}$, $I_{i\,\mathrm{juv}}^{\mathrm{r}}$ – are the elements of the matrices [I_{nb}^{gen}], [I_{nr}^{gen}] of integral quality estimates for the benchmark and real FDSM.

Let's find the norm of the matrix [$\delta I_n^{\mathrm{gen}}$] in the form [2]

$$\left\|\delta I_n^{\mathrm{gen}}\right\| = \max_{i\,\mathrm{juv}}\left|\delta I_{i\,j\mathrm{uv}}\right| \tag{4}$$

If the norm

$$\left\| \delta I_n^{\text{gen}} \right\| \le 0.05 \tag{5}$$

then, as established above, we can speak about the possibility of obtaining reliable results of flight research on a partially similar free flying model. Equality to zero of the norm $\| \delta I_n^{\text{gen}} \|$ is equivalent to the case of complete coincidence of motion parameters of the real and reference FDSM.

As an illustration of the developed theoretical foundations, the author has performed calculations to determine the functional capabilities of the FDSM of the KhAI-70 aircraft in real design practice. At the same time, the methodological framework did not exhibit excessive "sensitivity" to the combined deviations of the principal MoI of the actual free-flying model (I_{xm} = 18 %, I_{ym} = 2 %and I_{zm} = 6 %) from those of the reference FDSM. Although the values $\delta \underline{I}_{i\,\text{juv}}(\alpha_m) = -0.0492$ and $\delta \underline{I}_{i\,\text{juv}}(\nu_m) = -0.0303$ are sufficiently close to the limit value of 0.05 [11].

As a second example, the author considers the application of this approach to demonstrate the effect of changes in the MoI I_z on the angle of attack α during stepwise elevator deflection of the turbojet business-class aircraft Learjet-24 by the American company Swiss American Aviation Corporation. The calculations showed that the difference in the integral evaluation I_0 for the aircraft with the nominal (baseline) and 20 % increased value of the MoI I_z increased value of the MoI 2 %. The deviation of the MoI I_z by 20% seems too large for solving modeling problems, but this method is not sensitive to the specific control laws of the aircraft's ACS [11].

For the considered FDSMs and aircraft, the ACS control laws are formulated in such a way that they lead to rapid damping of external disturbances, and even significant deviations in the MoI have only a minor effect on the motion characteristics. Thus, the idea of comparing the integral performance measures of the ACS for the actual and reference FDSM is reasonable. However, there is a need for additional studies to define the metric for accepting or rejecting partial similarity to ensure reliable results across a wide range of changes in the MoI for the aircraft (aircraft or its FDSM) with specific ACS control laws. Therefore, there is a need for further studies, especially to determine the spin characteristics, which are more sensitive to the mass distribution of the aircraft.

The approach to similarity assessment in flight dynamics modeling of an aircraft on its FDSM, discussed above, to some extent assumed that the FDSM's motion is constantly under the influence of the ACS control laws of the FDSM (similar to the actual aircraft) or simulates the behavior of the aircraft pilot. Such assumptions can be attributed to stable and controllable motion of the aircraft (either the actual aircraft or its FDSM). However, even in ordinary flight practice, cases of aircraft system failures (including the ACS), loss of pilot effectiveness, and the aircraft entering critical or hazardous uncontrolled flight regimes are not uncommon. The study of these flight regimes, as well as entry into and recovery from them, is precisely the prerogative of creating FDSMs for flight research.

3 Analysis of the Functional Capabilities of Aircraft FDSM with a Failed ACS

Therefore, it is necessary to consider primarily those flight regimes that are described by mechanical relations and that have a direct correlation between the parameters of the FDSM and its motion characteristics in space. At the same time, it is necessary to consider A.A. Gukhman's fundamental analogy: two motions are similar if, when superimposing two video images of the same point in space and time over a given period, no external differences are observed, or the differences are negligible. Differences are considered negligible if the values of similar geometric, kinematic, or dynamic quantities that define the motion process do not exceed ± 5 % throughout the entire observation period [12].

It should be noted that the value of ± 5 %, as before, is insufficiently justified, but it is adopted in mechanics for assessing the similarity of the behavior of bodies or dynamic systems. Therefore, in conducting further research, it is necessary to provide additional justification for it.

Of course, the analysis of the influence of errors in determining the mass-inertial parameters (MIP) of the FDSM during the design stage on the assessment of their functional capabilities in case of ACS failure should have been carried out long ago. However, it so happened that the customers of aircraft FDSMs (constructed at the RIPFM) were often either satisfied with the accuracy of flight research, or they did not know what to request or demand, nor how to do it. The author's reflections that not all primary flight regimes were covered by checks at the early stages of creating aircraft FDSMs have only now been formalized, even though the theoretical foundations of motion have long been established in mechanics.

Thus, in the considered field of knowledge, two types of solid body motion are of interest: translational motion and rotational motion.

Translational motion is the mechanical motion of a system of points (of a perfectly solid body), in which a segment connecting any two points of the body, the shape and size of which do not change during motion, remains parallel to its position at any previous moment in time. In translational motion, all points of the body describe identical trajectories, undergo identical displacements, travel equal distances, and at each moment in time have equal velocities and accelerations [13].

The fundamental equation of translational motion dynamics [13]

scalar form

$$F = m \cdot a \tag{6}$$

vector form

$$\boldsymbol{F} = m \cdot \boldsymbol{a} \tag{7}$$

where $\boldsymbol{F}$ – the force causing the acceleration of the body (the cause), N; $\boldsymbol{a}$ – the acceleration acquired by the body (the effect), [m/s^2]; m– the mass of the body (a measure of inertia in translational motion), [kg].

Rotational motion is the motion of a solid body in which all its points describe circles whose centers lie on a single straight line, called the axis of rotation. Only the points lying on the axis of rotation do not describe circles. The axis of rotation passing through the center of mass (CM) of a solid body is called the central axis, while the MoI are referred to as principal [10].

scalar form

$$M = I \cdot \alpha \tag{8}$$

vector form

$$\boldsymbol{M} = I \cdot \boldsymbol{\alpha} \tag{9}$$

where $\boldsymbol{M}$ – moment of force imparting angular acceleration to the body (the cause), N m; $\boldsymbol{\alpha}$ – angular acceleration acquired by the body (the effect), [rad/s^2]; $\boldsymbol{I}$ – MoI of the body relative to the axis of rotation (a measure of the body's inertia in rotational motion), [kg m^2].

As before, we assume that the degree of similarity in modeling should be assessed by comparing the behavior of geometrically indistinguishable aircraft FDSMs: the actually designed model (hereinafter simply “real” although it may differ substantially from the truly real one) and the reference model (fully meeting the similarity criteria). For the purposes of this study (within the limits of low altitudes and flight speeds), the reference and real aircraft FDSMs will be considered as solid bodies.

It is not difficult to see that Eqs (6) and (8), as well as (7) and (9), are of the same type and exhibit similar cause-and-effect (dynamic–kinematic) relationships. Therefore, by conducting a thought experiment using one of the formulas, the results can be extended to the other as well.

Thus, if an identical force $\boldsymbol{F}$, equal in magnitude and direction, is applied at the same point to both the reference and the real FDSM aircraft in a state of rest, then, according to (6), the uniformly accelerated translational motions of both FDSMs can be described by the following equations:

$$F = m_m^{\text{ref}} \cdot a_m^{\text{ref}} \tag{10}$$

$$F = m_m^{\text{real}} \cdot a_m^{\text{real}} \tag{11}$$

where a_m^{ref}, a_m^{real} – the accelerations acquired by the reference and the real FDSM aircraft, respectively, [m/s^2]; m_m^{ref}, m_m^{real} – the masses of the reference and the real FDSM aircraft, kg.

Since the forces applied to the reference and the real FDSM aircraft are identical, then, when considering Eqs (10) and (11) together, it is not difficult to derive the conditions required to satisfy the similarity requirements in modeling

$$m_m^{\text{ref}} \cdot a_m^{\text{ref}} = m_m^{\text{real}} \cdot a_m^{\text{real}} \tag{12}$$

or

$$\frac{a_m^{\text{real}}}{a_m^{\text{ref}}} = \frac{m_m^{\text{ref}}}{m_m^{\text{real}}} \tag{13}$$

On the other hand, the similarity condition, which imposes a limit on the maximum discrepancy of accelerations, takes the form

$$\left| \frac{a_m^{\text{ref}} - a_m^{\text{real}}}{a_m^{\text{ref}}} \right| = \left| 1 - \frac{a_m^{\text{real}}}{a_m^{\text{ref}}} \right| \le \delta a_m \tag{14}$$

where δa_m – the maximum relative discrepancy of the accelerations of the reference and the real FDSM aircraft (as noted above, a value of 0.05 may be adopted for δa_m).

By substituting the ratios of accelerations from (13) into (14), we obtain

$$\left| 1 - \frac{m_m^{\text{ref}}}{m_m^{\text{real}}} \right| \le \delta a_m \tag{15}$$

or after simplifications, we obtain

$$\left| 1 - \frac{m_m^{\text{ref}}}{m_m^{\text{real}}} \right| = \left| \frac{m_m^{\text{real}} - m_m^{\text{ref}}}{m_m^{\text{real}}} \right| = \frac{\left| m_m^{\text{ref}} - m_m^{\text{real}} \right|}{\left| m_m^{\text{real}} \right|} \le \delta a_m \tag{16}$$

Condition (16) can be directly applied (provided that the mass values are known or specified) to evaluate the feasibility of modeling on the real FDSM.

According to the methodological approaches of error theory in the problem of calculating the errors in determining the MIP of the designed FDSM aircraft, absolute and relative errors are distinguished. If x^{true} denotes the true value, and x^{calc} – the computed approximate value of the quantity x, then the difference $x^{\text{true}} - x^{\text{calc}}$ is called the true absolute error, and $(x^{\text{true}} - x^{\text{calc}})/x^{\text{calc}}$ – is the true relative error in determining the quantity x during computation. The true value of the quantity x is uncertain, and therefore the corresponding errors are also indeterminate. However, it is possible to specify a limiting value of the true absolute error, i.e., a positive number Δx^{calc}, for which the following inequality holds [14]

$$x^{\text{calc}} - \Delta x^{\text{calc}} \le x^{\text{true}} \le x^{\text{calc}} + \Delta x^{\text{calc}} \tag{17}$$

The quantity Δx^{calc} is called the error limit, or the maximum absolute error in determining the quantity x during computation, while $\delta x^{\text{calc}} = \Delta x^{\text{calc}} / |\Delta x^{\text{calc}}|$ – during computation, while x during computation.

By assuming in our limiting case that $m_m^{\text{ref}} = m_m^{\text{true}}$ and $m_m^{\text{real}} = m_m^{\text{calc}}$, the left-hand side of (16), according to (17), can be rewritten in the form

$$\frac{\left|m_m^{\text{true}} - m_m^{\text{calc}}\right|}{\left|m_m^{\text{calc}}\right|} = \frac{\Delta m_m^{\text{calc}}}{\left|m_m^{\text{calc}}\right|} = \delta m_m^{\text{calc}} \tag{18}$$

and finally, we obtain the condition linking the maximum relative error in calculating the initial mass of the FDSM aircraft with the maximum relative discrepancy of the accelerations of the reference and the real FDSM aircraft.

$$\delta m_m^{\text{calc}} \le \delta a_m \tag{19}$$

If a value of $\delta a_m = 0.05$ is adopted (which is quite sufficient for the vast majority of mechanical problems), then condition (19) takes a form convenient for analysis

$$\delta m_m^{\text{calc}} \le 0.05 \tag{20}$$

Since the main equations of translational motion dynamics in scalar form (6) and of rotational motion dynamics in scalar form (8) are of the same type and exhibit similar cause-and-effect (dynamic–kinematic) relationships, as indicated above, by performing a thought experiment using one of the formulas, the results can be extended to the other. Therefore, if an identical torque M, equal in magnitude and direction, is applied at the same point to both the reference and the real FDSM aircraft at rest, then the condition linking the maximum relative error δI_m^{calc} in calculating the MoI about the rotation axis with the maximum relative discrepancy of the angular accelerations of the reference and the real FDSM, δa_{M}, is obtained:

$$\delta I_m^{\text{calc}} \le \delta \alpha_m \tag{21}$$

and relative to the axes of the OXYZ coordinate system with its origin at the CM of the FDSM:

$$\delta I_{jm}^{\text{calc}} \le \delta \alpha_m \ \left(j = x, y, z\right) \tag{22}$$

If a value of $\delta a_m = 0.05$ is adopted (as was done for δa_m, then condition (22) takes a form convenient for analysis

$$\delta I_{jm}^{\text{calc}} \le 0.05 \ \left(j = x, y, z\right) \tag{23}$$

In the above formulas, m_m^{calc} represents the calculated values of the initial mass of the FDSM m_m^{I}, m_m^{II}, m_m^{III}, while I_{jm}^{calc} represents the calculated values of the MoI of the FDSM I_m^{I}, I_m^{II}, I_m^{III} for three successive approximations at the stages of preliminary, conceptual, and detailed design.

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4 Validation of Theoretical Foundations for Assessing Functional Capabilities of Aircraft FDSMs with a Failed ACS

As a validation of the obtained conditions, we consider the results of the calculations of the mass-inertial parameters of the FDSM SLMT-10S-BP of the Su-27 aircraft (Fig. 2) for studying the stability and controllability of this aircraft over a wide range of angles of attack at speeds close to stall speeds [3, 11].

Thus, in the calculation of the initial mass of the FDSM

$$\delta m_m^{\mathrm{I}} = 0.2456 \quad \delta m_m^{\mathrm{II}} = 0.2176 \quad \delta m_m^{\mathrm{III}} = 0.041$$

and in the calculation of its axial MoI relative to the axes of the OXYZ coordinate system with its origin at the CM of the FDSM

$$\delta I_{xm}^{\mathrm{I}} = 0.3224 \quad \delta I_{xm}^{\mathrm{II}} = 0.115 \quad \delta I_{xm}^{\mathrm{III}} = 0.0649$$

$$\delta I_{ym}^{\mathrm{I}} = 0.303 \quad \delta I_{ym}^{\mathrm{II}} = 0.1082 \quad \delta I_{ym}^{\mathrm{III}} = 0.047$$

$$\delta I_{zm}^{\mathrm{I}} = 0.3071 \quad \delta I_{zm}^{\mathrm{II}} = 0.1083 \quad \delta I_{zm}^{\mathrm{III}} = 0.044$$

It is not difficult to notice that at the first two (least accurate) stages of FDSM design, condition (15) for the initial mass and condition (18) for the axial MoI of the SLMT-10S-BP are not satisfied, and achieving modeling similarity of flight regimes on such a real FDSM is not possible. However, at the third stage – the detailed design stage – these conditions are not satisfied only for the MoI relative to the longitudinal axis of the FDSM, OX. The insignificance of the deviation, $\delta I_{xm}^{\mathrm{III}} = 0.0649$ allowed, during the subsequent rearrangement of equipment, to satisfy condition (18) and achieve modeling similarity, as confirmed by the results of flight tests on the SLMT-10S-BP.

Similar positive design outcomes were achieved in the development of several FDSM aircraft created at the RIPFM.

Fig. 2 FDSM SLMT-10S-BP of the Su-27 aircraft on the ground launch rig

5 Conclusions

The author of this work has proposed two specialized approaches for assessing the similarity and functional capabilities of FDSM when modeling the flight dynamics of an aircraft. In the implementation of the first approach, the motion of the FDSM is considered to be continuously influenced by the control laws of the FDSM's ACS (similar to a full-scale aircraft) or simulates the behavior of the pilot of the aircraft itself. Such assumptions can be attributed to the stable and controllable motion of the aircraft (either the full-scale aircraft or its FDSM). However, even in normal flight operations, cases of aircraft failure (including the ACS), loss of pilot functionality, and the aircraft entering critical or hazardous uncontrolled flight regimes are not uncommon. The study of these flight regimes, as well as of entering or recovering from them, is precisely the prerogative of creating FDSM for flight testing and forms the basis of the second approach.

The study presents research aimed at the analysis and evaluation of the existing approach based on numerical simulation methods and the mathematical framework for investigating the quality of the ACS in the task of determining the functional capabilities of aircraft FDSM, as well as at the development of theoretical foundations for a new approach to assessing the design functional capabilities of an aircraft FDSM in the event of ACS failure. At the same time, the degree of similarity was determined by comparing the behavior of the actually designed and the reference FDSM.

For the first time, the second approach to assessing similarity in flight dynamics modeling of an aircraft using its FDSM has been considered: the theoretical foundations of translational and rotational motion have been applied; the influence of errors in determining the inertial parameters of aircraft FDSMs at the design stage on the assessment of their functional capabilities without considering the influence of the ACS has been analyzed; and the defining conditions have been formalized. As an approbation of the derived defining conditions, the results of calculations for the FDSM in real design practice have been considered.

In this way, both approaches to the assessment of similarity in aircraft flight dynamics simulation using the FDSM have been formally established. The differences are considered negligible if the values of the corresponding geometric, kinematic, or dynamic quantities that determine the motion of the real and reference FDSM, or a given norm, do not exceed ± 5 % throughout the entire observation period. It should be noted that the ± 5 % value is not sufficiently substantiated, but it has been adopted in mechanics for assessing the similarity of the behavior of bodies or dynamic systems. Therefore, further research is needed to provide additional justification for this value.

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