Advances in Military Technology
Vol. 20, No. 1, 2025, pp. 81-93
ISSN 1802-2308, eISSN 2533-4123
DOI 10.3849/aimt.01963

Assessment of the Defeating Effectiveness of Ballistic Missiles by Air Defense Systems

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The manuscript was received on 1 February 2025 and was accepted after revision for publication as a case study on 15 April 2025.

Abstract:

Combating tactical and short-range ballistic missiles in modern military conflicts is the most critical task of the air defense forces. The paper describes a method for assessing the effectiveness of hitting tactical and short-range ballistic missiles by an anti-aircraft missile system, which considers the time characteristics of the stages of the combat operation process of an anti-aircraft missile system. The anti-aircraft missile system functions as a part of the tactical missile defense system. The method is advisable to use when deciding to protect facilities from ballistic missile strikes.

Keywords:

ballistic missile, anti-aircraft missile system, tactical missile defense system, probability of hitting, probability distribution density of time characteristic

1 Introduction

Equipping the armies of many countries around the world with tactical and short-range ballistic missiles (BM) has led to their widespread use in military conflicts to destroy the most essential assets for both military and civilian use [1-3].

Combating tactical and short-range BM is the most critical task of the air defense forces of the defending country, as the use of BM causes significant damage to the country's assets [4]. However, this task is problematic because BMs are the most challenging and least vulnerable targets for anti-aircraft missile systems (AAMS) [3, 5].

The effectiveness of combating BM depends both on the availability of AAMS capable of hitting them and on the organization of defense of assets from their strikes, as well as on the level of training of combat personnel of AAMS, command post radar and command posts. Asset protection from BM strikes must be carried out based on the analysis of the results of assessing the effectiveness of their defeat by AAMS

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[6, 7]. For the assessment, it is necessary to use the appropriate methodological provisions.

2 Formulation of Problem and Setting Objective

For thorough assessing of effectiveness of air target destruction, it is necessary to consider the random processes that occur during the firing of an AAMS.

The paper [7] develops a probabilistic model for calculating BM defense effectiveness that can be used to determine the technical performance required for a BM defense system to meet specific defense objectives. The defense objective is stated as a certain probability that no warheads leak through the defense. In the paper [8], the mathematical model is designed to optimize the assets' defense from attacks by ballistic missiles. The model uses mathematical optimization and mixed integers linear programming solutions to reduce the damage against enemy attacks. The method for assessing the effectiveness of the air and missile defense system based on parametric diagrams is given in the article [9]. The parametric diagrams were used to evaluate the effectiveness of interception phase and calculate the successful probability of intercepting an enemy missile by AAMS. However, these models do not consider the random processes that occur during the combat operation process of an AAMS.

The article [10] considers the methodology based on the simulation model of the functioning process of the air defense mean when repelling the CM strike. The model simulates in detail all stages and operations of combat work on the air defense mean by time.

The considered methods sufficiently fully set out the main provisions for assessing the effectiveness of AAMS firing at aerodynamic targets, considering the combat operation process. However, the methods do not take into consideration the features of assessing the effectiveness of AAMS firing at a BM and the influence of the time characteristics of the stages of the combat operation process of an AAMS on the effectiveness of destroying a BM. These circumstances require further development of methodological provisions for assessing the effectiveness of hitting BM by AAMS.

The purpose of the article is to develop a methodology for assessing the effectiveness of AAMS firing at BM, which considers the time characteristics of the stages of the combat operation process of an AAMS.

3 Description of Methodology and Basic Mathematical Equations

3.1 Structure and States of Functioning of the Tactical Missile Defense System

The appropriate system-wide military-technical problem of ensuring the fight against BM is the timely detection of their launches and the issuance of target designations to AAMS. According to experts, a tactical missile defense system (TMDS) can be created to solve this problem [6]. Such a system should include means of warning of BM launches, their detection, tracking during flight, and destruction.

The prototype of the TMDS is the system created by the US Armed Forces during hostilities in the Persian Gulf with satellite-based means of detecting BM launches, data processing and transmission systems, and Patriot AAMS [11].

The TMDS can be considered an air defense system component. The TMDS, in particular universal AAMS, can simultaneously perform the air defense tasks of troops and assets in the general air defense system. However, combining BM launch detection devices into a single whole, their detection in the airspace, AAMS control, and fire destruction of the missile can be considered as a separate system – the TMDS, the structure of which is shown in Fig. 1.

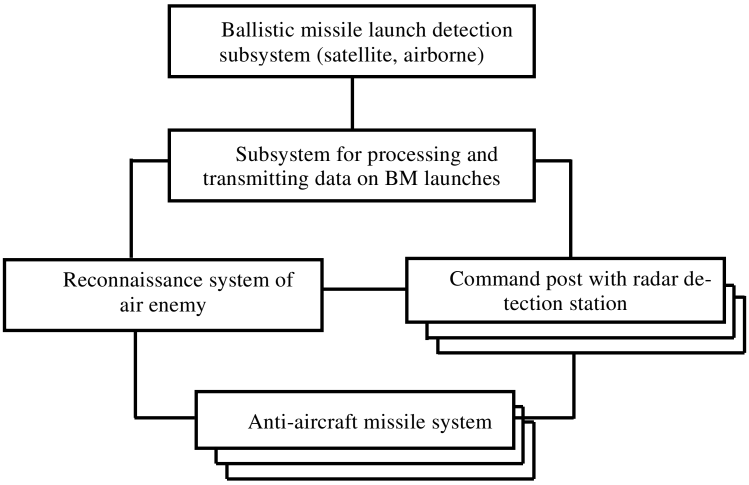

Fig. 1 Generalized structure of the tactical missile defense system

The effectiveness of defeating a BM by an AAMS must be assessed by considering the functioning processes of all TMDS elements.

During operation, the TMDS may be in the following states, which correspond to the stages of its functioning:

- E_1 – detection of BM launches,
- E_2 – transition of elements of the TMDS to readiness status,
- E_3 – detection of enemy's BM by an airspace reconnaissance system and issuance of target designation to the command post,
- E_4 – detection of a BM by the command post radar, issuance of target designation to the AAMS,
- E_5 – detection of a BM by an AAMS and capturing it for escort,
- E_6 – preparation and launch of an AAGM,
- E_7 – guidance of an AAGM on a BM,
- E_8 – defeating of a ballistic missile by an AAGM,
- E_9 – BM is not damaged.

The state transition graph of the TMDS is shown in Fig. 2. During the operation of the TMDS, the transition from one state to another is carried out sequentially. However, according to Fig. 2, other transition options are possible. BM detection can be carried out by the radar of the command post without target designation by the airspace reconnaissance system, i.e., the E_3 state is excluded. Also, target designation can be issued by the airspace reconnaissance system directly to the AAMS, i.e., the E_4 state is excluded.

Subsequently, the state transition graph of the TMDS (Fig. 2) is used not for decision-making but to construct a probabilistic scheme of the functioning of the TMDS according to the stages of its functioning, thereby assessing the probability of the

TMDS being in a certain state, which is used when determining the probability of hitting a BM.

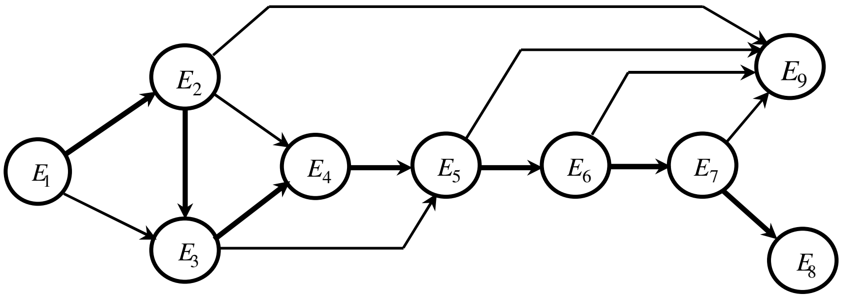

Fig. 2 The state transition graph of the tactical missile defense system

3.2 Construction of a Probabilistic Scheme of the Functioning of the Tactical Missile Defense System

The transition of the TMDS from one state to another is characterized by the distances to the BM and the corresponding time characteristics of the stages of its functioning. It is proposed to use the probability distribution densities of the time characteristics of the combat operation process, which correspond to the stages of the TMDS operation, to assess the effectiveness of defeating a BM. It is assumed that the probability distribution laws of the time characteristics of the combat operation process are close to Gaussian normal distribution.

The parameters of the probability distribution laws of the time characteristics of the TMDS combat operation process can be obtained using the standards [8]. As the mathematical expectation of each of them, the time value corresponding to the "good" rating can be chosen. The doubled range between the values corresponding to the "satisfactory" and "excellent" ratings is taken as six standard deviations.

The evaluation of the effectiveness of the AAMS firing is carried out for a specific trajectory of a BM, the parameters of which in time (slant range, horizontal range to the BM relative to the position of the AAMS, altitude, and flight speed) should be specified in a table. In this case, it is advisable to consider the following:

- BM trajectories can be optimal, hovering, and flat,
- the optimal trajectory is implemented at a certain launch angle of the BM and provides the greatest (maximum) range of its flight,
- BM trajectories above the optimal one are lofted, and those below are flat,
- the type of BM trajectory for assessing the effectiveness of an AAMS firing must be determined considering the distance from the starting position of the BM to the defended object,
- the parameters of the BM trajectory are determined by modeling its flight at a range corresponding to the distance to the strike object,
- the method uses a local rectangular coordinate system [12].

The firing effectiveness of the AAMS is assessed by the probability of hitting a BM (ς). The general expression for assessing the probability of hitting a BM, considering the stages of the TMDS, has the form

$$\varsigma = P_{\text{rel}} W_{\text{prl}} W_{\text{g}} W_{\text{def}} \tag{1}$$

where P_{rel} – the probability of reliable functioning of all elements of the TMDS;

W_{prl} – the conditional probability of successful completion of pre-launch stages of the TMDS;

W_{g} – the conditional probability of successful guidance of an AAGM;

W_{def} – the conditional probability of defeating a BM.

If n AAGM are fired at BM, then the probability W_{def} is determined

$$W_{\text{def}} = 1 - \left(1 - R_{\text{def}}\right)^{n} \tag{2}$$

where R_{def} – the conditional probability of hitting one BM by one AAGM.

Considering the randomness of state transitions of the TMDS (Fig. 2), a probabilistic scheme of its functioning during the firing of an AAMS at a BM was constructed, which is shown in Fig. 3. The diagram shows the distributions of time characteristics, and Tab. 1 gives the parameters of the laws of distribution of time characteristics of the combat operation process.

In addition to those given in Tab. 1, the following designations are adopted in the diagram:

τ_{ot} – the operational time of the TMDS (the flight time of a BM from the moment of its launch detection to the launch of an AAGM);

τ_{fip} – the flight time of an AAGM to the interception point of a BM;

τ_{ll} – the lead launch time of an AAGM;

$H_{\text{upb}}, H_{\text{lb}}$ – the upper and lower boundaries of the BM damage zone;

H_{req} – the required (set) BM damage altitude;

t_{req} – the flight time of a BM to the required BM engagement altitude.

When solving the problem of the rendezvous of an AAGM with a BM, it is assumed that the flight time of the AAGM to the rendezvous point τ_{fip} is equal to the travel time of the BM to the lead-in point τ_{ll} .

The average flight time of an AAGM to the interception point of a BM ($\overline{\tau}_{\text{fip}}$) and the standard deviation of this time (σ_{fip}) are determined by modeling the process of guiding an AAGM to the target (functioning of the AAGM control system).

3.3 Analytical Expressions for Evaluating the Effectiveness of an Anti-Aircraft Missile System Firing at a Ballistic Missile

According to the scheme in Fig. 3, the average flight times of a BM at the stages of the TMDS operation are determined by Eq. (3).

$$\left.\begin{aligned}
\overline{t}_{\text{ld}} &= \overline{\tau}_{\text{ld}} \\
\overline{t}_{\text{cr}} &= \overline{t}_{\text{ld}} + \overline{\tau}_{\text{cr}} \\
\overline{t}_{\text{dr}} &= \overline{t}_{\text{cr}} + \overline{\tau}_{\text{dr}} \\
\overline{t}_{\text{dcp}} &= \overline{t}_{\text{dr}} + \overline{\tau}_{\text{dcp}} \\
\overline{t}_{\text{de}} &= \overline{t}_{\text{dcp}} + \overline{\tau}_{\text{de}} \\
\overline{t}_{\text{pl}} &= \overline{t}_{\text{de}} + \overline{\tau}_{\text{pl}} \\
\overline{t}_{\text{ll}} &= \overline{t}_{\text{pl}} + \overline{\tau}_{\text{ll}}
\end{aligned}\right\} \tag{3}$$

Tab. 1 Parameters of the laws of distribution of time characteristics

Stages	Content of stages (time characteristics)	Parameters of the laws of distribution		Average flight time of BM by stages
		Mathematical expectation	The mean squared deviation	
E_1	BM launch detection	$\overline{\tau}_{\mathrm{ld}}$	σ_{ld}	$\overline{t}_{\mathrm{ld}}$
E_2	transition of elements of the TMDS to readiness status	$\overline{\tau}_{\mathrm{cr}}$	σ_{cr}	$\overline{t}_{\mathrm{cr}}$
E_3	detection of a BM by an airspace reconnaissance system and issuance of target designation to the command post	$\overline{\tau}_{\mathrm{dr}}$	σ_{dr}	$\overline{t}_{\mathrm{dr}}$
E_4	detection of a BM by the command post radar, issuance of target designation to the AAMS	$\overline{\tau}_{\mathrm{dcp}}$	σ_{dcp}	$\overline{t}_{\mathrm{dcp}}$
E_5	detection of a BM by an AAMS and capturing it for escort	$\overline{\tau}_{\mathrm{de}}$	σ_{de}	$\overline{t}_{\mathrm{de}}$
E_6	preparation and launch of an AAGM	$\overline{\tau}_{\mathrm{pl}}$	σ_{pl}	$\overline{t}_{\mathrm{pl}}$
E_7	guidance of an AAGM on a BM	$\overline{\tau}_{\mathrm{ll}}$	σ_{ll}	$\overline{t}_{\mathrm{ll}}$

The specified time characteristics correspond to the BM trajectory parameters, which are defined in a table.

The mathematical expectation of the operating time of a TBDM is determined by

$$\overline{\tau}_{\mathrm{ot}} = \overline{\tau}_{\mathrm{cr}} + \overline{\tau}_{\mathrm{dr}} + \overline{\tau}_{\mathrm{dcp}} + \overline{\tau}_{\mathrm{de}} + \overline{\tau}_{\mathrm{pl}} \tag{4}$$

The standard deviation of the operating time is determined by

$$\sigma_{\mathrm{ot}} = \sqrt{\sigma_{\mathrm{cr}}^2 + \sigma_{\mathrm{dr}}^2 + \sigma_{\mathrm{dcp}}^2 + \sigma_{\mathrm{de}}^2 + \sigma_{\mathrm{pl}}^2} \tag{5}$$

Time t_{req} is determined in accordance with the required height of the BM H_{req}. In this case, the horizontal range to the BM d_{hr} must be within the AAMS firing range.

The methodology assumes that the mathematical expectation of the lead time is $\overline{\tau}_{\mathrm{ll}} = \overline{\tau}_{\mathrm{fip}}$. The standard deviation of the lead time is $\sigma_{\mathrm{ll}} = \sigma_{\mathrm{lt}}$, where σ_{lt} is the mean square error in determining the lead time (instrument error).

To assess the probability of successful completion of pre-launch stages, compositions of probability distribution laws of time characteristics are used. The composition of the distribution laws refers to finding the total law of several random variables. According to the probabilistic scheme of the TMDS, to assess the indicator W_{prl}, compositions of distribution laws are calculated by Eq. (6).

Then the probability of successful completion of the pre-launch stages of the TMDS (the probability of timely launch of an AAGM) is determined by the formula Eq. (7).

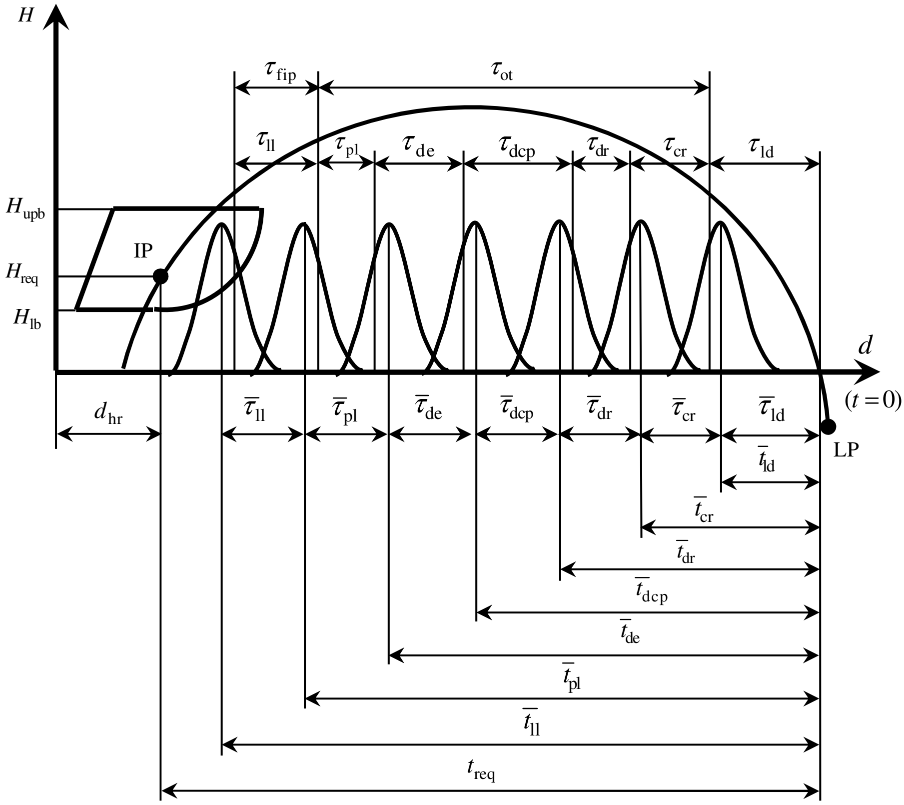

Fig. 3 Probabilistic scheme of the functioning of the tactical missile defense system

$$\left.\begin{aligned}
R_1\left(t_{\text{req}}\right) &= \Phi\left[\frac{t_{\text{req}}-\left(\overline{\tau}_{\text{ll}}+\overline{\tau}_{\text{ot}}\right)-\overline{t}_{\text{ld}}}{\sqrt{\sigma_{\text{ll}}^2+\sigma_{\text{ot}}^2+\sigma_{\text{ld}}^2}}\right] \\
R_2\left(t_{\text{req}}\right) &= \Phi\left[\frac{t_{\text{req}}-\left(\overline{\tau}_{\text{ll}}+\overline{\tau}_{\text{pl}}+\overline{\tau}_{\text{de}}+\overline{\tau}_{\text{dcp}}+\overline{\tau}_{\text{dr}}\right)-\overline{t}_{\text{cr}}}{\sqrt{\sigma_{\text{ll}}^2+\sigma_{\text{pl}}^2+\sigma_{\text{de}}^2+\sigma_{\text{dcp}}^2+\sigma_{\text{dr}}^2+\sigma_{\text{cr}}^2}}\right] \\
R_3\left(t_{\text{req}}\right) &= \Phi\left[\frac{t_{\text{req}}-\left(\overline{\tau}_{\text{ll}}+\overline{\tau}_{\text{pl}}+\overline{\tau}_{\text{de}}+\overline{\tau}_{\text{dcp}}\right)-\overline{t}_{\text{dr}}}{\sqrt{\sigma_{\text{ll}}^2+\sigma_{\text{pl}}^2+\sigma_{\text{de}}^2+\sigma_{\text{dcp}}^2+\sigma_{\text{dr}}^2}}\right] \\
R_4\left(t_{\text{req}}\right) &= \Phi\left[\frac{t_{\text{req}}-\left(\overline{\tau}_{\text{ll}}+\overline{\tau}_{\text{pl}}+\overline{\tau}_{\text{de}}\right)-\overline{t}_{\text{dcp}}}{\sqrt{\sigma_{\text{ll}}^2+\sigma_{\text{pl}}^2+\sigma_{\text{de}}^2+\sigma_{\text{dcp}}^2}}\right] \\
R_5\left(t_{\text{req}}\right) &= \Phi\left[\frac{t_{\text{req}}-\left(\overline{\tau}_{\text{ll}}+\overline{\tau}_{\text{pl}}\right)-\overline{t}_{\text{de}}}{\sqrt{\sigma_{\text{ll}}^2+\sigma_{\text{pl}}^2+\sigma_{\text{de}}^2}}\right]
\end{aligned}\right\} \tag{6}$$

where $\Phi(...)$ is the normal distribution function.

$$W_{\text{prl}} = R_1\left(t_{\text{req}}\right)R_2\left(t_{\text{req}}\right)R_3\left(t_{\text{req}}\right)R_4\left(t_{\text{req}}\right)R_5\left(t_{\text{req}}\right) \tag{7}$$

The conditional probability of successful guidance of an AAGM W_g is quantitatively characterized by the probability of an encounter between an AAGM and a BM within the strike zone of an AAMS. The probability W_g is determined by the formula

$$W_g = \Phi\left(\frac{t_{\text{fl}} - \overline{t}_{\text{dcl}}}{\sigma_{\text{dcl}}}\right) - \Phi\left(\frac{t_{\text{fup}} - \overline{t}_{\text{dcl}}}{\sigma_{\text{dcl}}}\right) \tag{8}$$

where t_{fup} – the flight time of a BM to the upper boundary of the destruction zone of an AAMS;

t_{fl} – the flight time of a BM to the lower boundary of the destruction zone of an AAMS;

$\overline{t}_{\text{dcl}}$ – the mathematical expectation of the conditional law of distribution of the flight time of a BM to the rendezvous point (given the launch time of an AAGM $\overline{t}_{\text{pl}}$);

σ_{dcl} – the standard deviation.

Time value $\overline{t}_{\text{dcl}}$ is equal to

$$\overline{t}_{\text{dcl}} = \overline{t}_{\text{pl}} + \tau_{\text{fip}} \tag{9}$$

The standard deviation of time is σ_{dcl}

$$\sigma_{\text{dcl}} = \sqrt{\sigma_{\text{fip}}^2 + \sigma_{\text{lt}}^2} \tag{10}$$

Using Eq. (8) to estimate the conditional probability of successful guidance of an AAGM involves finding the meeting point with a BM within the strike zone of the AAMS, which means that the following condition must be met:

$$t_{\text{fup}} \le \overline{t}_{\text{dcl}} \le t_{\text{fl}} \tag{11}$$

If $\overline{t}_{\text{dcl}} < t_{\text{fup}}$ (expectation of an AAGM launch), it is accepted

$$\overline{t}_{\text{dcl}} = t_{\text{fup}} + \Delta\tau_{\text{ldt}} \tag{12}$$

where $\Delta\tau_{\text{ldt}}$ – the launch delay time of an AAGM.

The probability of hitting a BM by a warhead of an AAGM is determined by an experimental-theoretical method. First, experiments are conducted to determine characteristics of hitting a BM, characteristics of the warhead of an AAGM, and a fuse. The errors of pointing an AAGM at a BM are determined by modeling. Subsequently, the following are determined: laws of distribution of guidance errors; coordinate law of hitting a BM; law of fuse activation. Using these laws, the probabilities of hitting a BM by an AAGM are calculated [13]. These probabilities of hitting are confirmed during firing at real BM during field tests of AAMS.

In the methodology for assessing the effectiveness of AAMS firing at BM, the conditional probability of hitting a BM by an AAGM R_{def} is given in the initial data and is considered constant within the area of impact of the AAMS.

The probability of failure-free operation of elements of the TMDS is estimated using a separate methodology and is specified in the initial data.

The structural diagram of the methodology for assessing the effectiveness of firing an AAMS against BM is shown in Fig. 4.

When assessing the effectiveness of firing at BM, it is necessary to take into account the capabilities of the airspace reconnaissance system, the command post radar, and the AAMS to detect them. It may happen that the means of the TMDS will be brought into combat readiness earlier than the BM can be detected. Therefore, it is necessary to adjust the time characteristics $\bar{t}_{dr}$, $\bar{t}_{dcp}$, $\bar{t}_{de}$.

To do this, the corresponding data D^*_{dr}, D^*_{dcp}, D^*_{de} are determined from the table of parameters of the BM trajectory for the time characteristics $\bar{t}_{dr}$, $\bar{t}_{dcp}$, $\bar{t}_{de}$. If these ranges exceed the detection ranges D_{dr}, D_{dcp}, D_{de}, specified in the original data, then the time characteristics $\bar{t}_{dr}$, $\bar{t}_{dcp}$, $\bar{t}_{de}$ are determined from the table by the ranges D_{dr}, D_{dcp}, D_{de}.

The presented methodology is developed in accordance with the probabilistic scheme (Fig. 3), which provides for the sequential implementation of all stages of the functioning of the TMDS.

In case when the sequence of performing stages of combat operation process changes (excluded states E_3 or E_4), components ($\bar{\tau}_{dr}$, σ_{dr}, $\bar{t}_{dr}$) or ($\bar{\tau}_{dcp}$, σ_{dcp}, $\bar{t}_{dcp}$) in the Eq. (6) will be equal to zero.

As an example, Tab. 2 presents the generalized characteristics of BM, which are obtained from the analysis of the characteristics of existing tactical and short-range BM [14].

Tab. 2 General characteristics of short-range BM

Maximum firing range [km]	Maximum flight altitude [km]	Flight time at maximum range [min]	Maximum flight speed [m/s]	Average horizontal flight speed [m/s]
150	60	3.0	1 100	830
300	90	5.0	1 300	1 000
600	170	8.5	1 700	1 200

3.4 Example

To assess the effectiveness of hitting a BM, averaged (approximate) initial data were adopted, which corresponds to modern AAMS used to destroy BM:

- upper and lower boundaries of the BM damage zone H_{upb} = 25 km, H_{lb} = 1 km,
- required height of impact for a BM H_{req} = 15 km,
- horizontal range to the interception point of a BM d_{hr} = 15 km,
- slant range to the interception point of a BM d_{sl} = 21.2 km,
- conditional probability of hitting a BM by one AAGM R_{def} = 0.7,
- number of AAGM for firing at BM n = 2,
- average anti-aircraft guided missile flight speed V_{fs} = 1250 m/s,
- flight time of an AAGM to the interception point $\bar{\tau}_{fip}$ = 17 s,
- reliability of operation P_{rel} = 0.98,

DOI 10.3849/aimt.01963

- mathematical expectations of the laws of distribution of time characteristics: $\overline{\tau}_{\mathrm{ld}} = 60$ s, $\overline{\tau}_{\mathrm{cr}} = 30$ s, $\overline{\tau}_{\mathrm{dr}} = 10$ s, $\overline{\tau}_{\mathrm{dcp}} = 6$ s, $\overline{\tau}_{\mathrm{de}} = 6$ s, $\overline{\tau}_{\mathrm{pl}} = 15$ s,
- average quadratic deviations: $\sigma_{\mathrm{ld}} = 10$ s, $\sigma_{\mathrm{cr}} = 10$ s, $\sigma_{\mathrm{dr}} = 3$ s, $\sigma_{\mathrm{dcp}} = 2$ s, $\sigma_{\mathrm{dc}} = 2$ s, $\sigma_{\mathrm{pl}} = 5$ s, $\sigma_{\mathrm{fep}} = 2$ s, $\sigma_{\mathrm{lt}} = 2$ s,
- BM detection range: $D_{\mathrm{dr}} = 90$ km, $D_{\mathrm{dcp}} = 80$ km, $D_{\mathrm{de}} = 70$ km.

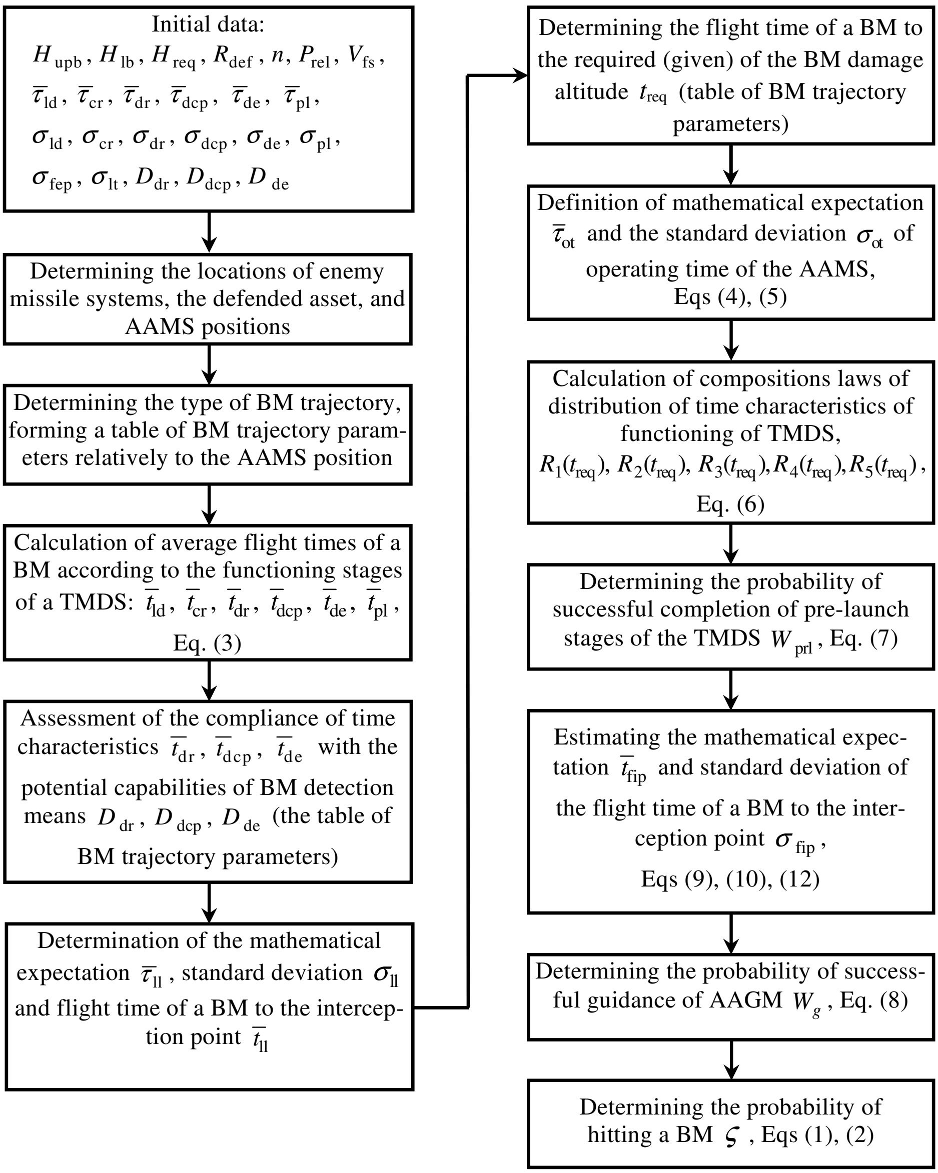

Fig. 4 Block diagram of methodology for assessing the effectiveness of firing an anti-aircraft missile system at a ballistic missile

The results of determining the average flight times of a BM by the stages of the TMDS operation are given in Tab. 3.

Tab. 3 Average flight times of BM by stages of the TMDS operation

BM firing range [km]	$\bar{t}_{\mathrm{ld}}$ [s]	$\bar{t}_{\mathrm{cr}}$ [s]	$\bar{t}_{\mathrm{dr}}$ [s]	$\bar{t}_{\mathrm{dcp}}$ [s]	$\bar{t}_{\mathrm{de}}$ [s]	$\bar{t}_{\mathrm{pl}}$ [s]	$\bar{t}_{\mathrm{ll}}$ [s]
150	60	90	100	106	112	127	144
300	60	90	225	231	237	252	269
600	60	90	448	454	460	475	492

When determining the average flight time of a BM before its detection by an air-space reconnaissance system $\bar{t}_{\mathrm{dr}}$ for BM firing ranges of 300 km and 600 km, waiting times of 115 s and 338 s were considered.

The results of calculating the probabilities of successful completion of pre-launch stages, successful guidance of an AAGM and defeat of a BM are given in Tab. 4.

Tab. 4 Probabilities of successful completion of pre-launch stages, successful guidance of an AAGM and defeat of a BM

Maximum firing range of the BM [km]	t_{req} [s]	Compositions of distribution laws					W_{prl}	t_{fup} [s]	t_{fl} [s]	$\bar{t}_{\mathrm{dcl}}$ [s]	W_g	ς
		R_1	R_2	R_3	R_4	R_5						
150	162	0.92	0.97	1.0	1.0	1.0	0.89	157	179	144	0.96	0.76
300	285	0.85	0.91	0.99	1.0	1.0	0.77	275	299	269	0.96	0.66
600	497	0.62	0.66	0.78	0.79	0.81	0.20	489	509	492	0.85	0.15

The too low probability of hitting a BM (ς = 0.15) to the required height H_{req} = 15 km when firing a BM at a range of 600 km is due to its high flight speed. However, for the required height H_{req} = 5 km, the probability of hitting a BM is quite high (ς = 0.83). This indicates that an AAMS with the characteristics adopted in the initial data of the methodology can be quite successfully used to combat BM as part of the TMDS.

4 Discussion

The developed methodology considers the distribution of the probabilities of time characteristics of the stages of the AAMS combat operation process, which are random and allocated by the Gaussian normal distribution.

To evaluate the effectiveness of hitting a BM, it is proposed to use the probabilistic scheme of functioning of TMDS. This scheme reflects the sequence of considering the distribution laws of time characteristics of the process of TMDS combat work in the stages of its functioning. On the basis of the use of the compositions of the probability distribution of temporal characteristics according to the probabilistic scheme

(Fig. 3), analytical dependencies Eq. (6) were obtained to determine the probability of successful performance of pre-start stages of functioning of TMDS (W_{prl}), which influence the effectiveness of hitting the BM (Tab. 4) the most.

Unlike existing approaches, the effectiveness of hitting a BM by AAMS is estimated by its functioning in the TMDS, taking into consideration the time characteristics of the combat operation process in accordance with the states, as shown in Fig. 2. Also, the obtained analytical dependencies allow to evaluate the effectiveness of hitting a BM by separate AAMS.

The technique makes it possible to evaluate the impact of the time characteristics of the process of TMDS combat work on the efficiency of hitting a BM. This can be used to form tactical and technical specifications for the modernization of existing or the development of new AAMS samples.

The results of evaluating the effectiveness of hitting the BM of a specific type (firing range, type of trajectory) can be used to research the issues of determining the number of AAGM to guarantee them being destroyed.

The developed technique is focused on evaluating the effectiveness of the AAMS hitting tactical and small-range BM, which is a limitation of its use.

5 Conclusions

An analytical methodology for assessing the effectiveness of defeating operational and tactical ballistic missiles of an AAMS has been developed. The use of an AAMS as part of a TMDS, which ensures the detection of BM launches and the timely bringing of the means into combat readiness, has been considered.

The effectiveness of defeating a BM is defined as the product of the probability of failure-free operation of the elements of the TMDS, the conditional probabilities of successful completion of pre-launch stages, successful guidance of an AAGM, and defeat of a BM by the AAGM.

Considering the specificity of the trajectory of a BM, that is, that the flight altitude of a BM exceeds the upper limit of the zone of attack of an AAMS, the methodology, unlike existing approaches, uses the probability distribution densities of the time characteristics of the combat operation process. These distribution densities correspond to the stages of the TMDS operation when determining the probability of successful completion of the pre-launch stages. At the same time, the potential capabilities of BM detection means are taken into account.

The methodology allows us to assess the impact of the capabilities of BM detection means and the time characteristics of combat operations on the probability of hitting a BM.

The procedure for applying the methodology is shown in the example of assessing the effectiveness of defeating a BM with a firing range of 150, 300, and 600 km.

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