



Synthesis of Algorithm to Change Guidance Law for Air-to-Air Missiles Intercept Separating Targets

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Abstract:

This paper presents the results of summarizing the algorithm of target selection and changing guidance law applied for medium and long-range air-to-air intercept missiles to ensure the ability to detect and distinguish the group-separating target, which in turn improving the efficiency in tracking and terminating the selected targets. The proposed algorithm reduces the miss distance to a minimal value while ensuring two primary requirements, including distinguishing each target in the group by Doppler frequency and identifying the flight direction of the target to track and destroy.

Keywords:

algorithm of choosing optimal guidance laws, separating target, supermaneuverable target, optimal proportional pre-emptive navigation law

1 Introduction

There have been substantial research and development of missile guidance laws [1-7], each of which has its own optimal criteria according to energy, error, fast acting, and ability to distinguish targets. Currently, there is no guidance law that can optimally meet all the criteria as stated, especially for all target audiences. A practically rising problem is to change the method of guidance even when the missile has been launched and is following a group of targets at a long distance. However, when the missile travels sufficiently close to the targets, the individual targets in the group become separate because of the distinguishing ability of the homing head radar. It is clear that the intercept missile needs an optimal guidance law according to its ability to distinguish separate targets at a long distance. In the case of close distance, it is necessary to have another guidance law according to the guiding accuracy after choosing a target to destroy.

For “air-to-air” intercept missiles, guidance laws are based on proportional

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guidance law PN [7], which has different variants depending on the selected optimization criteria. The advantage of the PN guidance law in [2, 4, 7] is that the missile's trajectory has a relatively small initial curvature and miss distance, resulting in a high probability of destroying the selected target, while the guidance law APN can intercept maneuvering targets [1, 5, 6]. However, a drawback of the PN and APN guidance laws is that their approaching trajectory is relatively straight, which is a disadvantage connected with detecting each target in the group at a long distance separately, especially the opponent's countering missiles called the separating target. This disadvantage is the inherent property of the two guidance laws, which is caused by the inhomogeneity between the observation angle and the Doppler frequency received from the target. The disadvantage arises when the fluctuation of the closing velocity (V_c) estimated by the received Doppler frequency and the fluctuation of the angular value of the targets in the group cancel each other, leading to the indistinguishability of each target in the group. That is the reason why there is a need for an optimal guidance law according to the criterion of target discrimination in the group for long-range interceptor missiles. As discussed in [8-14], the authors claim that when using the conventional radar pulse method to detect groups of targets at long distances, it is impossible to distinguish them in the group separately. For the missile homing head radars to distinguish each target individually in a group at a distance of more than 30 km, the guidance law must create a more curved trajectory than the conventional ones. Then, the reflected signal from each target will have a different Doppler frequency shift, and their spectra are also different because of the secondary modulation effect. By narrow band Doppler frequency filtering and secondary spectrum filtering, we can distinguish each target separately and identify them in the group.

Starting from that idea, this paper presents a method of synthesizing a new guidance law, which is optimized according to the Doppler frequency discriminant criterion by the narrow band filter based on the PN law. The proposed guidance law resolves the limitations of the homing head radars in terms of the ability to distinguish by distance (detection pulse width) and the ability to distinguish by angle (receiving antenna wing width) when the missile is tracking a group of targets or a single separating target. Although the guidance law in this paper is optimal for target discrimination, it is not optimal in terms of guidance accuracy because it has a larger orbital curvature than PN or APN ones. This leads to the second problem that needs to be solved, which is to synthesize a conversion algorithm of guidance law for "air-to-air" missiles after correctly detecting the target to be destroyed in the group.

Each of the aforementioned problems are presented and solved in the remaining sections of this paper.

2 Solution of the Problems

2.1 *Synthesis of Optimal Guidance Laws for Air-to-Air Missiles to Distinguish a Target or a Group of Targets*

For the general situation, the problem of optimal control law synthesis uses linear models defined in state space, which can be expressed by the following differential equation:

$$\dot{x} = f[x(t), u(t), t] \tag{1}$$

where $x(t)$ is the state vector of dimension n_x , $u(t)$ is the input (control) vector of dimension n_u . To synthesize the optimal control law based on the normal acceleration of the missile, it is necessary to minimize the following quality function [15].

$$J = \int_0^{t_k} \Phi_T[x(t), u(t), t] dt + \int_0^{t_k} \Phi_K[x(t_k), u(t_k), t_k] dt \tag{2}$$

where $\Phi_T[...]$, $\Phi_K[...]$ – the general expression under the integral sign corresponds to the instantaneous and terminal components. The control will be optimal if J is consistently minimal in the time interval $[0; t_k]$.

Based on the above problem synthesis of the guidance law for air-to-air missiles to the group target, the kinematic model of the AAM intercept missile must clearly take into account the relative displacement between the interceptor-AAM and the targets in the group when the antenna beam of the homing head on the missile is oriented towards the group target. In other words, it is necessary to orient the interceptor's velocity vector to be inclined at an angle other than the orbital tilt angle of the conventional PN guidance law so that it can be obtained separately from radial velocities (closing velocity V_c) corresponding to each target in the group. This can ensure the ability of the homing head to distinguish each target according to the Doppler frequency (F_D) received in the reflected signal and identify them through secondary modulation of the reflected signal.

The kinematic correlation describing the relative motion between an interceptor missile and a target in the group is illustrated in Fig. 1, which shows that the interceptor missile is guided to the target according to the PN [7] and according to the proposed guidance law.

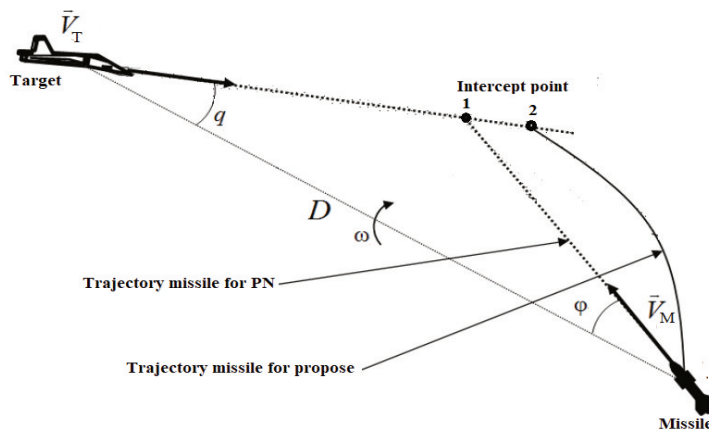


Fig. 1 Dynamic geometry of relative motion between the missile and the target when using the guiding law by the PN and according to the proposed guiding law

where \vec{V}_T, \vec{V}_M – corresponds to the target and missile velocity vectors; D – the distance between target-missile; ω – the angular velocity of the line of sight missile-target in the horizontal plane; φ – the missile lead angle (the angle between the missile velocity

vector and the line of sight); q – the aspect angle (the angle between the target velocity vector and the line-of-sight).

According to [7], the equation describing the dependence of the angular velocity of the line of sight on the normal acceleration of the missile is expressed as follows:

$$\dot{\omega} = -\frac{2\dot{D}}{D}\omega + \frac{1}{D}(W_T - W_M); \omega(0) = \omega_0 \quad (3)$$

where W_T , W_M represent the normal acceleration of the target and the missile in the horizontal plane, respectively. To synthesize the optimal guidance law for interceptor missiles to meet the requirements of target discriminating in the group by Doppler frequency, we need to add Eq. (3) to the differential equations that describe direct control of the interceptor missile's trajectory as follows:

$$\dot{\omega} = 0; \omega_r(0) = \frac{\lambda}{2\Delta L_{\min}}\omega_q \quad (4)$$

where ω_r – the angular velocity of the line-of-sight required to produce the deviation of the orbital tilt angle missile trajectory, ω_q – the angular velocity of the antenna lobe of the missile's homing head, [rad/s], ΔL_{\min} – the smallest resolution according to the distance of the homing head's radar, [m], λ – the wavelength of the detection pulse generator of the homing head's radar, [m].

From the analysis of Fig. 1, the expression for the angular velocity of the line-of-sight missile-target can be found as follows:

$$\omega = \frac{V_M \sin \varphi - V_T \sin q}{D} \quad (5)$$

From Eq. (5), the opening angle of the target recognition cone can be determined by the formula:

$$q = \arcsin \frac{V_M \sin \varphi - \omega D}{V_T} \approx \arcsin \frac{V_M \varphi - \omega D}{V_T} \quad (6)$$

Differentiate (6) concerning time, we have:

$$\dot{q} = \frac{V_M \dot{\varphi} - \dot{\omega} D - \omega \dot{D}}{\sqrt{V_T^2 - (V_M \varphi - \omega D)^2}} \quad (7)$$

Substitute Eq. (3) into (7), we get:

$$\dot{q} = \frac{\omega \dot{D} + V_M \dot{\varphi} + W_M - W_T}{\sqrt{V_T^2 - (V_M \varphi - \omega D)^2}} \quad (8)$$

$$\dot{q} = C(\omega \dot{D} + V_M \dot{\varphi} + W_M - W_T) \quad (9)$$

in which

$$C = \frac{1}{\sqrt{V_T^2 - (V_M \varphi - \omega D)^2}} \quad (10)$$

From Eqs (4), (9), and (10) together with Eq. (3), it is possible to determine a general kinetic model, which is the basis for synthesizing the optimal guidance law for missiles to intercept the group target (or separating target). This guidance law

allows the radar of the homing head to have the best conditions for radio observation, discrimination, identification of targets in the group, and detection of maneuvering targets (separating from the group). When the separating target is detected, the intercepting missile will be switched to track and destroy this target. In general, we have the following system of differential equations:

$$\left. \begin{aligned} \dot{\omega} &= -\frac{2\dot{D}_i}{D_i}\omega_i + \frac{1}{D_i}(W_M - W_{iT}) + \xi_\omega \\ \omega_i(0) &= \omega_{0i}; \dot{\omega}_{ir} = 0; \omega_{ir}(0) = \frac{\lambda}{2\Delta L_{\min}}\omega_q \\ \dot{q}_i &= C(\omega_i D_i; V_M \varphi_i + W_M - W_{iT}) \\ q_i(0) &= q_{0i}; \dot{q}_{ir} = 0; q_{ir}(0) = \text{const.} \end{aligned} \right\} \quad (11)$$

in which: D_i – presents the relative distance between the missile and the i^{th} target in the group, W_M , W_{iT} – the relatively normal acceleration of the missile and the i^{th} target, ω_i – the angular velocity of the i^{th} line-of-sight missile-target, ξ_ω – Gaussian white noise with a given one-sided spectral density, ω_{ir} – the required normal component for the angular velocity of the i^{th} line-of-sight missile-target, q_i – the radio observation angle for the secondary modulated signal, reflected from the i^{th} target to the radar of the homing head, q_{ir} – the required radio observation angle for the second modulated signal, reflected from the target to the radar of the homing head.

In the process of guiding a missile, the main control parameter is the normal acceleration of the interceptor. When synthesizing the guiding law, the normal acceleration must be optimal. It allows to guarantee of the conditions, related to the radio observation of the mobile radar for the reflected signals from the targets in the group, and to satisfy the requirement of minimum miss distance during the guided process. The control vector U is represented as a scalar as follows:

$$U = W_M \quad (12)$$

The required angular velocity of the line-of-sight missile-target (ω_r) and the required radio observation angle (q_r) (for the homing head's radar to have the ability to distinguish the reflected signal by Doppler frequency) are already required parameters for the missile guidance system. They form a vector of required phase coordinates of the form:

$$X_T = [\omega_r \quad q_r]^T \quad (13)$$

It follows that the controlled parameters in the guidance system are ω and q , and then the vector of controlled phase coordinates is expressed as:

$$X_Y = [\omega \quad q]^T \quad (14)$$

Since the control parameter is the missile's normal acceleration (W_M), and the controlled parameters are (ω and q), the efficiency matrix of the control signals (B_Y^T) will gather the coefficients related to (W_M). The differential Eqs (3); (4); (7); (9) and (10) describe the change of the controlled parameters as (ω and q). From the optimal kinematic model (11), the effective matrix of the control signals will have the form:

$$\mathbf{B}_Y^T = \begin{bmatrix} -1 \\ D \quad C \end{bmatrix} \quad (15)$$

When considering the unit size of the control vector Eq. (12) and the dimension of matrices Eqs (13)-(15), we can determine the penalty matrices for the tracking accuracy Q and the value of control signals K as follows.

$$\mathbf{Q}_Y^T = \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \quad (16)$$

$$K = k \quad (17)$$

where q_{11} and q_{22} are penalty matrix coefficients.

The result of minimizing the optimal criterion Eq. (1) is as follows:

$$J = M_Y \left\{ \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix}^T \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix} + \int_0^{t_k} W_M^2 k dt \right\} \quad (18)$$

The optimal control law which minimizes the criterion Eq. (1), is determined by the following formula:

$$U(t) = K^{-1} \mathbf{B}_Y^T Q [X_T(t) - X_Y(t)] \quad (19)$$

This is the control signal and the solution of the Bellman equation [15]. Thus, after solving Eq. (19) and taking into account Eqs (12)-(16), we can get the optimal control law for the normal acceleration of the AAM:

$$\left. \begin{aligned} W_{\text{OPPN}} &= \frac{1}{k} \begin{bmatrix} -1 \\ D \\ C \end{bmatrix}^T \begin{bmatrix} q_{11} & 0 \\ 0 & q_{22} \end{bmatrix} \begin{bmatrix} \omega_r - \omega \\ q_r - q \end{bmatrix} \\ W_{\text{OPPN}} &= -\frac{q_{11}}{kD} (\omega_r - \omega) + \frac{Cq_{22}}{k} (q_r - q) \end{aligned} \right\} \quad (20)$$

Eq. (20) is called the Optimal Proportional Pre-emptive Navigation Law (OPPN) for a missile intercepting a group target or separating target. Based on the guidance law expression (20), it is noticeable that the first part of the law permits distinguishing the targets separately in the group according to the Doppler frequency in the reflected signal received by the homing head's radar. In the second part, the missile is guided along a trajectory where its velocity vector always makes a required deviation angle with the line-of-sight missile-target, which is necessary for sustained observation of the reflected signals containing their own Doppler frequency and secondary modulation, serving the problem of identifying the type of the target in the group.

2.2 Synthesis Algorithm to Select the Separating Target and Change the Guidance Law for Interceptor Missile

Let us assume that the control system of the interceptor missile is always capable of switching between two guidance laws. Of course, when switching from one guidance law to another, the system will have to pay the price by the oscillation of the transition. However, for an automatic control system with sufficient stability reserve, this process only occurs for a time exactly equal to the transient time of the system, which is negligible compared to the remaining self-guiding time.

The optimization criterion here is not only for using the new guidance law OPPN to detect the separating target, but also for the solution of retargeting and converting to guidance law PN to follow the target.

The basis for selecting the target in the group and changing the guiding law is the condition of observing the target group by radar of the homing head, and information from the sensors on the missile compartment such as accelerometers, angle sensor and angular velocity, target rangefinder, Doppler frequency closing velocity meter and on-board digital computer.

The target selection algorithm and change of guidance law will be synthesized in the condition that the interceptor missile is initially guided by the OPPN method. So, all targets in the group can be observed based on Doppler frequency analysis and identified by secondary modulation effect in the spectrum of the reflected signal received by the homing heads radar. In the process of observing and analyzing targets, the algorithm can decide to choose one of the targets to destroy and automatically switch to the appropriate guidance law (PN or APN) with the following two main conditions:

- the target is a separating missile according to the results of spectrum identification and count the number of targets,
- the information of the separating target is “not towards us” from the maneuverable detector.

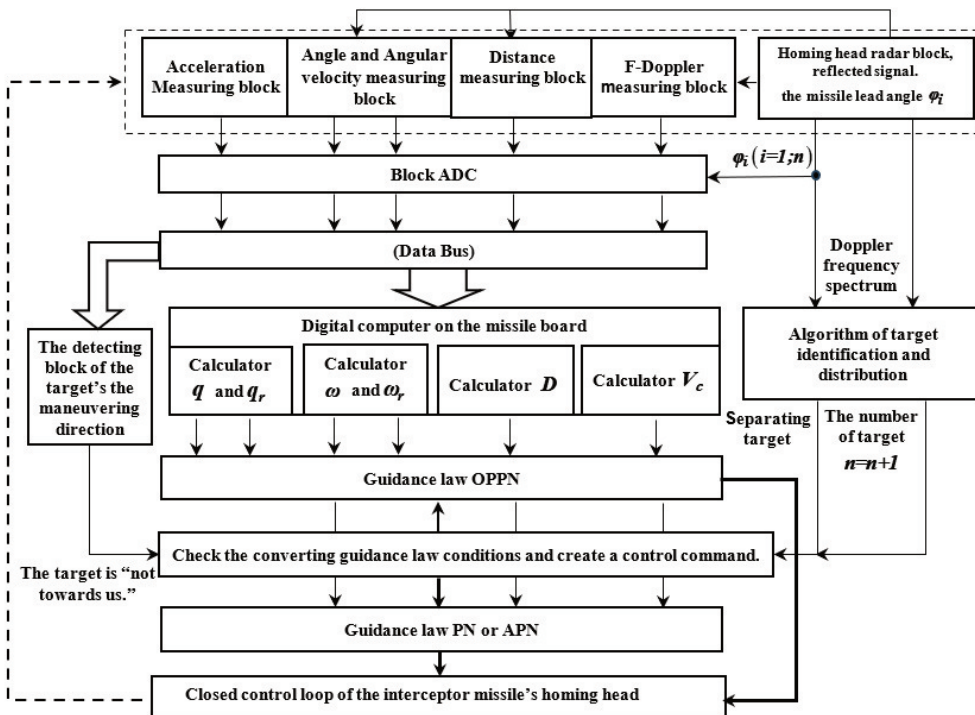


Fig. 2 Organizational diagram of an algorithm to select targets and change the guiding law for missiles to intercept separating target

With the assumption that the algorithm counts the number of targets in the group and detects the target direction as known (because it is presented separately in another paper), this paper mainly focuses on the algorithm to select the separating targets and change the guidance law to destroy the target.

The organizational structure diagram of the algorithm for selecting the target and changing the guidance law is shown in Fig. 2.

A flowchart of an algorithm for targets selecting and changing guidance law for interceptor missiles to separate targets is shown in Fig. 3. Notes in this algorithm are:

- (i) is the numeral of a target in a group of (n) targets,
- control commands are generated only for an initially selected target. In each control command loop (λ_{OPPN}), the radar antenna lobe of the homing head scans the targets in the group in turn from 1 to n to get information about the distance, Doppler frequency, and missile lead angle in the horizontal plane to make calculation data.

According to the normalized diagram of the algorithm to select the guidance law, the initial law applied to the interceptor missiles is OPPN, which is switched to PN [7] (or APN [1] if the target is highly maneuverable). When two conditions are satisfied respectively: The number of targets in the group increases and the separating target is “Not moving towards us”.

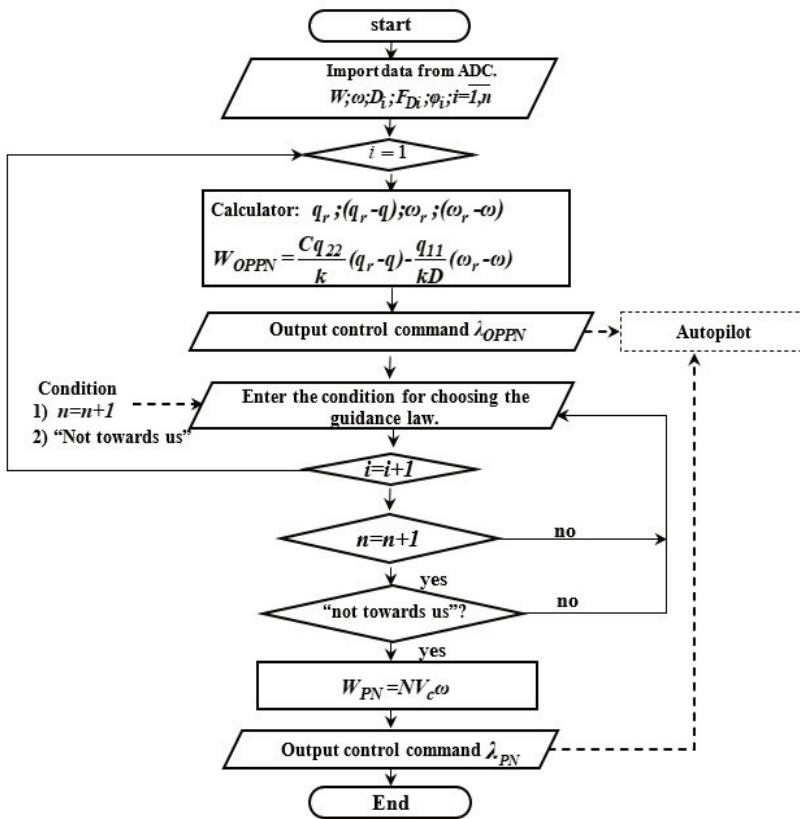


Fig. 3 Flowchart of an algorithm to choose the guidance law for missiles to intercept the separating target

3 Results and Discussions

To demonstrate the efficiency of the guidance law of selecting algorithms, several simulations have been performed with the scenarios related to the state change of the separating targets. Simulation conditions (air situation) on the vertical plane of the two guidance laws OPPN and PN [7] are the same. The investigations are on the characteristics of the interceptor missile's trajectory, miss distance, and reflection angle according to both guidance laws when intercepting a target. Further studies are on the efficiency of the guidance law selecting algorithm using the guidance law OPPN for detecting separated target. The simulation parameters are as below:

Missile parameters:

- initial velocity: $V_M = 1\,000$ m/s,
- horizontal distance: 0 km,
- height: 0 km,
- required observation angle: 20° .

Target parameters:

- initial velocity: $V_T = 350$ m/s,
- horizontal distance: 30 km,
- height: 15 km,
- orbital tilt angle: 20° .

Angular velocity of the antenna lobe of the missile homing head: $\omega_q = 10$ rad/s.

The smallest resolution according to the distance of the homing head radar: $\Delta L_{\min} = 150$ m. The wavelength of the detection pulse of the homing head radar: $\lambda = 0.03$ m.

3.1 Comparative Survey of Quality of the Guidance Laws by PN and OPPN for the Same Single Target

Comment 1: Based on the simulation results, the following comments and assessments can be drawn:

- for the same target variant, the missile trajectory (Fig. 4) guided by the PN method is straighter than that of OPPN, which is consistent with the result that the instantaneous miss distance (Fig. 5) of PN decreases faster than OPPN, although both methods have the miss distance decreasing to zero in the vicinity of the intersection point.

- the radio observation angles (Fig. 6) for the signal reflected from the target carrying the Doppler frequency and the secondary modulation signal in the two methods have the law of decreasing over time. However, the OPPN method decreases more slowly than the PN method, which allows the missiles guided by OPPN to have a longer target observation time for discrimination.

Thus, when intercepting a single target, the trajectory of the missile and the miss distance according to the PN method is always better than that of OPPN. However, in the case of a group target, the OPPN method is more beneficial in terms of setting the radio signal observation angle and observation time, as stated in the research problem. That is the reason why the author wants to combine guidance law by stages to build an optimal algorithm according to both discriminatory ability and accuracy for interceptor missiles.

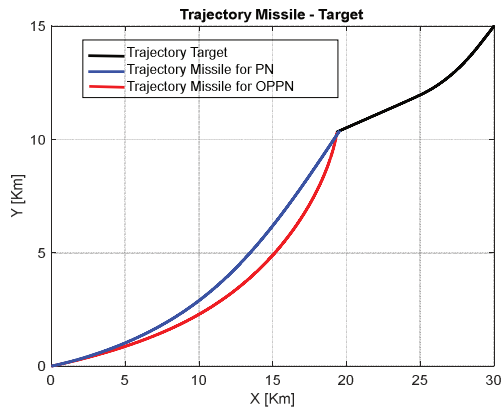


Fig. 4 Trajectory of the intercept missile and target

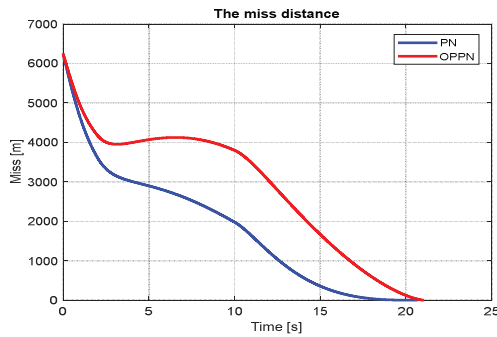


Fig. 5 Graph of the miss distance

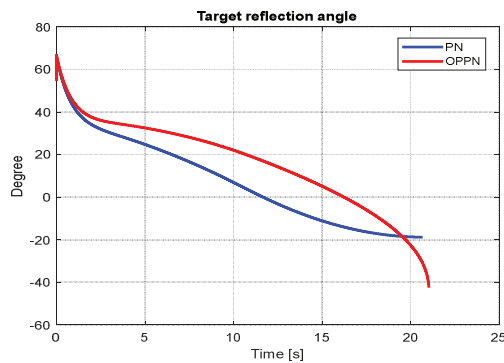


Fig. 6 Graph of change of target reflection angle

3.2 Evaluate the Effectiveness of the OPPN Guidance Law for the Separating Target

a) Results of Doppler frequency signal spectrum from targets

In Fig. 7, the Doppler signal spectrum reflected the carrier target (1), and in Fig. 8, the Doppler signal spectrum reflected both the carrier target (1) and the separating target

(2). The first condition in the algorithm of selecting the target and converting to guidance law is that the number of the group separating targets increases.

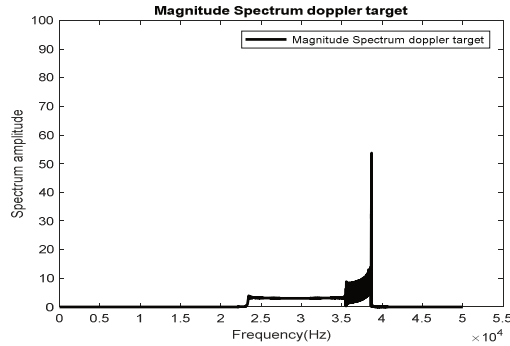


Fig. 7 Doppler Spectrum signal reflected from an original target

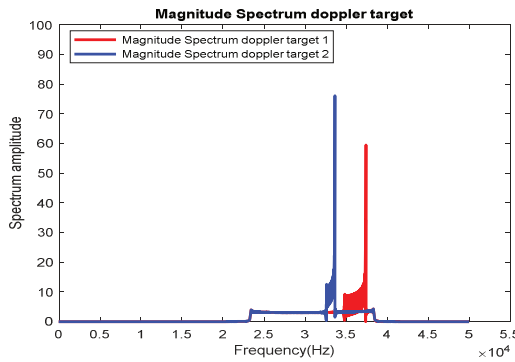


Fig. 8 Doppler Spectrum signal reflected from separating targets

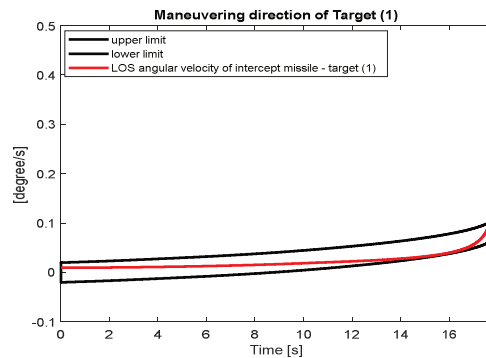


Fig. 9 The assessment result of the maneuvering direction of target (1)

b) Results evaluate the maneuvering direction of 2 targets in the group

The algorithm to detect maneuvering direction of the separating targets is presented in [16] by the authors of this paper, whose results are depicted in Figs 9 and 10. In Fig. 9, the maneuvering detector shows that target (1) is heading toward the interceptor missile, and the angular velocity line of sight is within the confidence limit. As shown

in Fig. 10, target (2) could be a group separating missile that is heading to the interceptor airplane group instead of the interceptor missile because the line-of-sight angular velocity is out of the range of confidence limits. This is a dangerous target which needs to be destroyed. The flight direction of the separating target is the second condition of the algorithm to select the target and change the guidance law. After determining the number of targets increased (target 2) and its direction is “Not towards us”, the interceptor missile can switch to the PN guidance law to destroy the target (2).

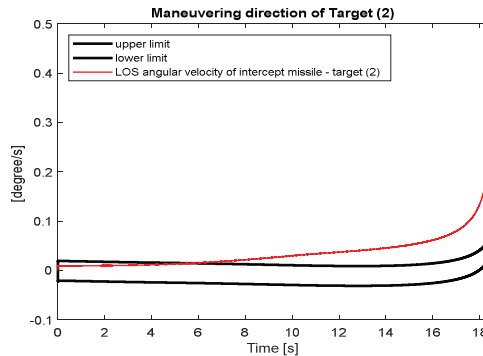


Fig. 10 The assessment result of the maneuvering direction of target (2)

3.3 Evaluate the Effectiveness of the Algorithm in Change the Guidance Law for Interceptor Missiles

a) In case the separating target flies at the same velocity and after 2 s changes in normal acceleration

Fig. 11 describes the trajectory of the intercepting missile and target. Velocity and normal acceleration of the target (1) and target (2) are illustrated in Fig. 12.

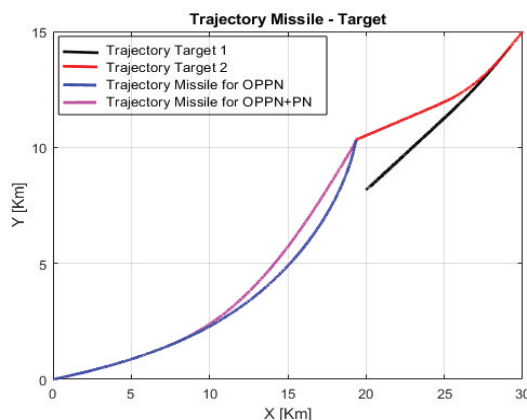


Fig. 11 Trajectory of the intercept missile and target

The results for the changes in target reflection angle and the miss distance for case a) are shown in Fig. 13 and Fig. 14 as follows.

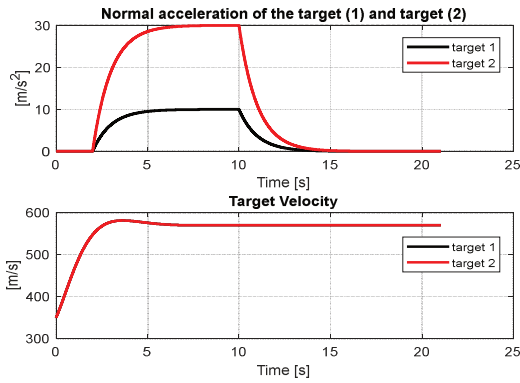


Fig. 12 Velocity and normal acceleration of the target (1) and target (2)

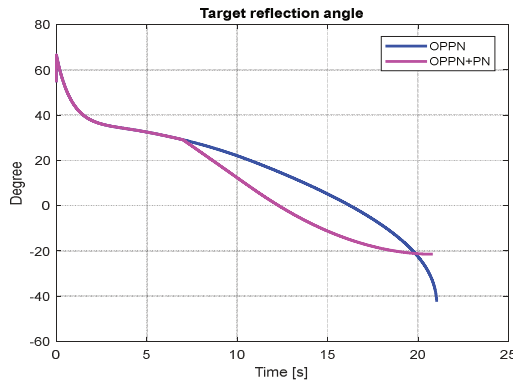


Fig. 13 Graph of change of target reflection angle

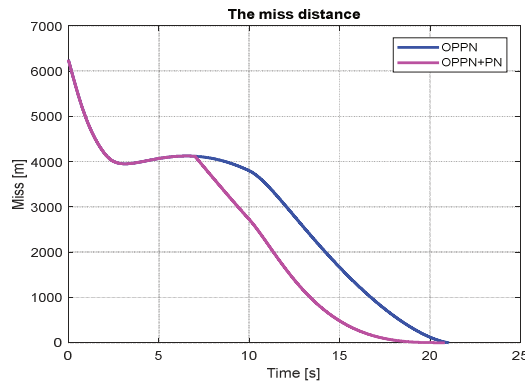


Fig. 14 Graph of the miss distance

b) In case the separating target simultaneously changes velocity and normal acceleration after 2 s

In this case, the trajectory of the intercepting missile and the target is depicted in Fig. 15, while Fig. 16 describes the velocity and normal acceleration for both targets.

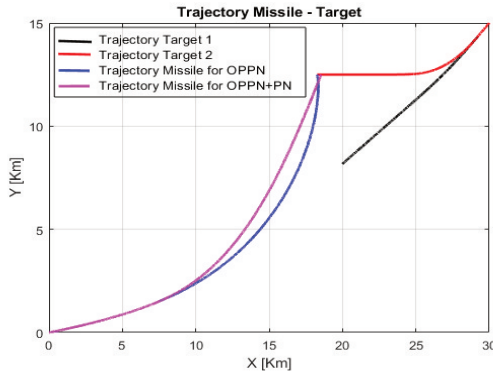


Fig. 15 Trajectory of the intercept missile and target

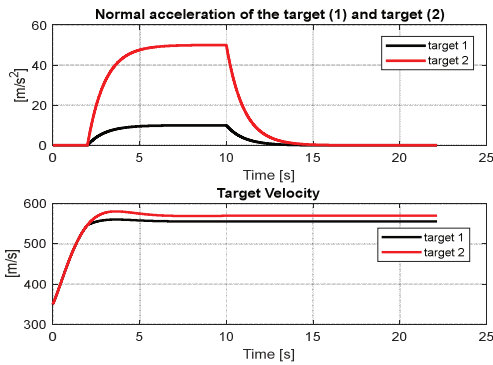


Fig. 16 Velocity and normal acceleration of the target (1) and target (2)

The results for the changes in the miss distance and target reflection angle for case b) are shown in Figs 17 and 18 as follows.

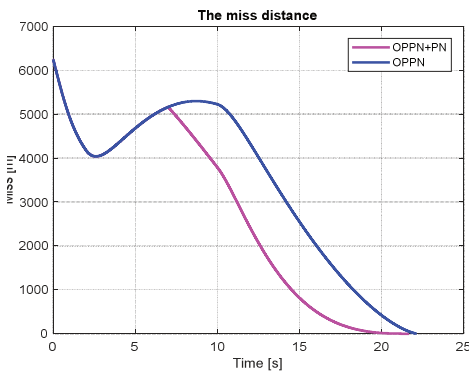


Fig. 17 Graph of the miss distance

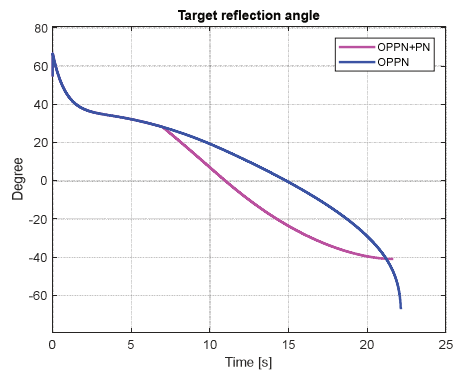


Fig. 18 Graph of change of target reflection angle

Comment 2: Through the simulation results of the target selection algorithm and conversion method for the problem of tracking to the separating target for two cases

a) and b), we see that the trajectory curvature of the interceptor missile does not produce any mutations when converting from OPPN to PN. The trajectory after using the converting method is more straight forward than the one using the guidance law OPPN. Therefore, through the results of Figs 14 and 17, we see that the miss distance in both graphs decreased to zero, but the miss distance when using the guidance law selection algorithm decreased faster than when using the guidance law by OPPN. The results of Figs 13 and 18 show that using the OPPN guidance law initially enables to observe the separating target; after completing target detection, the guidance law can be switched to PN. Thus, the use of the algorithm to select the target and convert the guidance law after detecting the separating target is better than using the algorithm OPPN according to both criteria of discriminating the separating target and minimizing the miss distance. That is, the combat effectiveness of the interceptor missile is enhanced.

4 Conclusion

The article has analyzed and synthesized an algorithm to select the targets in the group and changed the guidance laws for medium and long-range “air-to-air” intercept missiles when intercepting a target that was a group of aircraft, or an aircraft that launched a missile which is the same type towards the interceptor group. The missile is called a separating target, which is considered a supermaneuverability and especially dangerous class of targets. Through the survey results of the optimal OPPN and PN guidance law, and the algorithm of selecting the target and converting the guidance law, we clearly see the process of intercepting the targets in the group (or separating the group) at a long distance. Initially, intercept missiles must use OPPN guidance law to detect numbers of the targets in the group and each separating target in the group. After the step of detecting and distinguishing targets, the process of selecting a high-risk target for tracking and destruction is done by the algorithm of selecting targets and converting guidance law according to two conditions. The targets increase in number (by Doppler frequency spectrum analysis) and detect the target’s maneuvering direction from the maneuvering detector.

The simulation results have proved that the proposed algorithm has shown its optimal properties according to two consecutive criteria: the ability to distinguish targets at a long distance and the guiding accuracy at the end stage. This is also a requirement to improve the combat effectiveness of medium and long-range “air-to-air” interceptors.

References

- [1] LIU, Y., K. LI, L. CHEN, and Y.J.A.A. LIANG. Novel Augmented Proportional Navigation Guidance Law for Mid-Range Autonomous Rendezvous. *Acta Astronautica*, 2019, **162**, pp. 526-535. DOI 10.1016/j.actaastro.2019.05.031.
- [2] YANG, Z., W. JIANG, X. LIU and Z. QI. Predictive Control of Aircraft Control Systems for Maneuverable Target. *Journal of the Franklin Institute*, 2018, **355**(18), pp. 9036-9052. DOI 10.1016/j.jfranklin.2016.10.001.
- [3] RYOO, C.-K., H. CHO, and M.-J. TAHK. Optimal Guidance Laws with Terminal Impact Angle Constraint. *Journal of Guidance, Control, and Dynamics*, 2005, **28**(4), pp. 724-732. DOI 10.2514/1.8392.

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- [4] SHUKLA, U. S. and P.R. MAHAPATRA. The Proportional Navigation Dilemma-Pure or True. *IEEE Transactions on Aerospace and Electronic Systems*, 1990, **26**(2), pp. 382-392. DOI 10.1109/7.53445.
- [5] SIOURIS, G.M. *Missile Guidance and Control Systems*. New York: Springer, 2004. ISBN 0-387-00726-1.
- [6] VATHSAL, S. and A.K. SARKAR. Current Trends in Tactical Missile Guidance. *Defence Science Journal*, 2005, **55**(3), pp. 265-280. DOI 10.14429/DSJ.55.1991.
- [7] ZARCHAN, P. *Tactical and Strategic Missile Guidance*. 7th ed. Reston: American Institute of Aeronautics and Astronautics, 2019. ISBN 1-62410-584-X.
- [8] BOGDANOV, A.V., A.A. LOBANOV, A.A. KUCHIN and V.A. GOLUBENKO. A Method of Air Targets Complex Recognition in Air-Based Pulse-Doppler Radar at the Stage of Primary Signals Processing. *Journal of Siberian Federal University*, 2020, **13**(3), pp. 311-327. DOI 10.17516/1999-494X-0223.
- [9] VASILYEV, O.V., A.V. BOGDANOV, R.A. POTAPOV and A.S. SITNIKOV. The Air Targets Recognition-s at the Aviation Radars under the Prolonged Coherent Signals Processing (in Russian). *Journal Radioengineering*, 2012, **10**, pp. 131-136.
- [10] BOGDANOV, A.V., S.S. KOROTKOV, A.A. KUCHIN, V.N. BONDAREV and I.V. LYUTIKOV. The Concept of Recognition of Air Targets in the Aviation Radar Complex. *Journal of Siberian Federal University*, 2016, **9**(3), pp. 319-331. DOI 10.17516/1999-494X-2016-9-3-319-331.
- [11] BOGDANOV, A.V., O.V. VASIL'EV, V.A. GOLUBENKO and S.M. MANYASHIN. Method of Constructing Dynamic Models of Radial Velocities and Accelerations of Two Air Targets Flying in a Close Battle Formation. *Journal of Computer and Systems Sciences International*, 2007, **46**, pp. 636-645. DOI 10.1134/S1064230707040144.
- [12] ZAMARAEV, V.V., A.S. KUTUZOV, I.V. LYUTIKOV and D.V. MALCEV. Method of Constructing a Line Secretive Exit the Aircraft at a Given Point of the Detection Area Surveillance Radar Based on the Spectrum Analysis of the Doppler Frequency of the Received Signal. *Journal of Siberian Federal University*, 2021, **14**(3), pp. 281-291. DOI 10.17516/1999-494X-0309.
- [13] BOGDANOV, A.V., S.A. GORBUNOV, A.A. KUCHIN and S.A. SHPORTKO. Accuracy and Probabilistic Characteristics of the Helicopter Flight Mode Recognition Algorithm in an Airborne Radar System. *Journal of Siberian Federal University*, 2018, **11**(3), pp. 358-370. DOI 10.17516/1999-494X-0048.
- [14] SKRYPNIK, O.N. *Radio Navigation Systems for Airports and Airways*. Berlin: Springer, 2019. ISBN 981-13-7201-2.
- [15] LEVINE, W. Optimal Control Theory: An Introduction. *IEEE Transactions on Automatic Control*, 1972, **17**(3), pp. 423-429. DOI 10.1109/TAC.1972.1100008.
- [16] TUAN, N.T., V.H. TIEN, D.V. MINH and N.T. THU TRANG. Synthesis of the Optimal Algorithm to Detect the Maneuvering Direction of the Supermaneuverable Separating Target. *12th International Conference on Control, Automation and Information Sciences (ICCAIS)*, 2023.