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Evaluating the Effectiveness of Assets Protection by Air Defense Means from Cruise Missiles Strikes

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Abstract:

Air and sea-based cruise missiles are the most dangerous means of air attack. Local air defense is organized to protect assets from these strikes. This takes into account the possible directions of cruise missiles strikes and the effectiveness of assets protection. The effectiveness of assets protection from cruise missiles strikes is assessed by the probability of their preservation, which is determined using the developed methodology based on a simulation model of air defense. The method allows taking into consideration the impact of the peculiarities of the cruise missiles usage and the peculiarities of the air defense functioning on the effectiveness of assets protection, in repelling cruise missiles strikes. The application of the method is shown in an example.

Keywords:

air defense means, assets, cruise missiles, probability of assets preservation, simulation

1 Introduction

In the military conflicts of recent years, the usage of air and sea-based cruise missiles (CM) [1, 2] has been playing a significant role in achieving the goal of hostilities belongs to. The suddenness of usage, high targeting accuracy, powerful warheads, low radar visibility, and the ability to fly at extremely low altitudes characterize CM as the effective means of armed struggle and a difficult target for air defense (AD). The before-mentioned features of the cruise missiles usage significantly affect the functioning of air defense means [1-6]. Therefore, there is a need to study an important and urgent task to assess the effectiveness of assets protection by air defense means (ADm) from the cruise missiles strikes.

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2 Formulation of Problem and Setting Objective

Many scientific papers are dedicated to evaluating the effectiveness of AD of assets and troops. First of all, it is necessary to note the monograph [4, 5], in which the analytical-stochastic model of anti-air battle of anti-aircraft missile troops grouping is considered. Mathematical expectation of the number of damaged air targets by anti-aircraft missile troops group was accepted as an indicator of the effectiveness of AD in the model. However, such an indicator does not fully characterize the effectiveness of the assets protection, namely their preservation. In addition, the analytical-stochastic model does not take into account the peculiarities of the usage of CM.

The works [6, 7] consider the application of queuing theory methods to assess the effectiveness of repelling CM by antiaircraft missile systems (AAMs). In this case, the efficiency indicator is also a mathematical expectation of the number of destroyed targets and the peculiarities of the usage of CM are not considered here as well.

Article [8] uses the method of queuing theory with refusals of estimating the mathematical expectation of the number of targets that can be destroyed by AAMs of various types.

In monograph [6], the main provisions of construction of a simulation statistical model of a multi-channel AAMs during the reflection of CM strike are considered. The model creates deterministic flight trajectories of CM, which does not allow taking into account the uncertainty of the usage of CM in assessing the effectiveness of the assets protection. Moreover, the application possibilities of the model are limited to the simulation of an anti-aircraft missile system.

The monograph [9] presents a method for estimating the mathematical expectation of the number of destroyed targets by a formation of anti-aircraft missile troops. This method is based on the usage of the coefficients of participation of AAMs in repelling the strikes of air attack means, combat readiness of AAMs, the effectiveness of the command and control system, the impact of low altitudes, and maneuvers of air targets.

Thus, the methods in proceedings [4-9] mainly assess the firepower of AAMs and anti-aircraft missile troops grouping that is characterized by a mathematical expectation of the number of destroyed air targets from the strike. At the same time, in accordance with the main purpose of AD, to assess the effectiveness of assets protection from the cruise missiles strikes, it is advisable to use such an indicator as the probability of assets preservation. Besides, these methods do not fully take into account the uncertainty of the usage of CM.

The purpose of the article is to develop guidelines for the evaluating of effectiveness of assets protection by air defense means from CM strikes.

3 Methodology Description and Basic Mathematical Equations

To protect important assets from CM strikes, several anti-aircraft missile battalions (AAMb) or several anti-aircraft missile batteries (AAMbt) are usually assigned, i.e. local protection of assets is organized. The formation of the battle order of AAMb is carried out taking into account the results of forecasting the direction of CM strikes. This forecasting is carried out in accordance with the well-known operating principles of the CM control system [6, 10, 11].

For guidance at the end of the flight, CM are equipped with optical-electronic systems. In these systems, optically visible landmarks (objects or areas of terrain) within a radius of 10-15 km from the target are used to correct the flight of CM. It can be assumed that the attack by CM will most likely be made from the direction in which there is a greater number of landmarks for correcting their flight (Fig. 1).

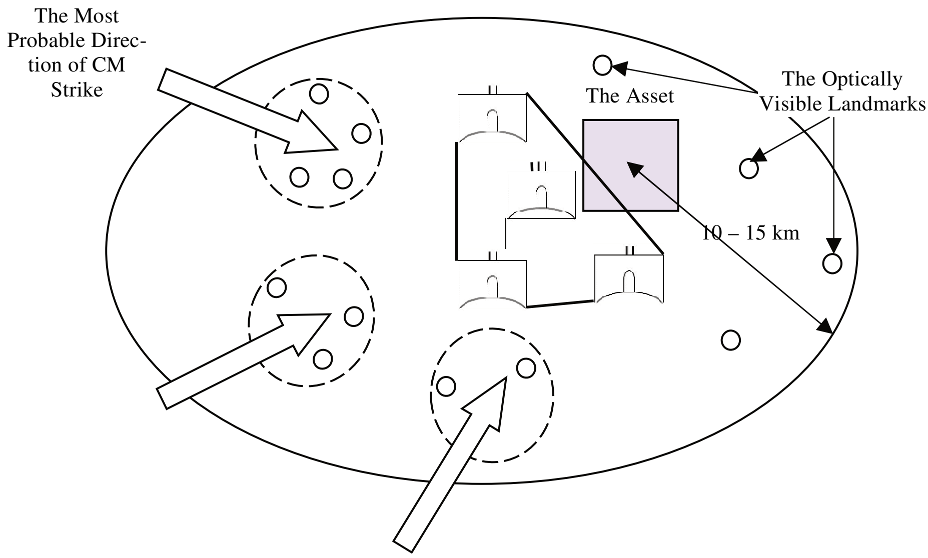

Fig. 1 Determining the direction of Cruise Missiles Strike

To evaluate the effectiveness of assets protection from CM strikes, the following indicators were suggested to use: M_{des} – the mathematical expectation of the relative number of CM destroyed by ADm; P_{pres} – the probability of assets preservation from CM strikes; $P_{\text{pres}}^{\text{app}}$ – the appointed probability of assets preservation from CM strikes.

Currently, methods based on the application of analytical-stochastic and simulation modeling methods are widely used to assess the effectiveness of AD of troops and assets [4, 6]. Simulation models use the principle of copying the simulated process. Simulation models are distinguished by the relatively simple possibility of taking into account complex formalized factors that affect the process of using ADm [4]. To consider stochastic uncertainty in simulation models, a statistical method is used when calculating indicators. In this way, through simulation modeling, realizations of random events are obtained, on the basis of which indicators are calculated. In such models, as a rule, the processes of functioning of radar reconnaissance of aerial targets, control and defeat are described with varying degrees of detail. This determines the expediency of using simulation models for evaluating the effectiveness of assets protection by ADm from CM strikes.

When simulating the process of the functioning of ADm, it is necessary to predict the flight routes of CM taking into account their uncertainty. For this purpose, it is proposed to use the method of randomly changing the flight directions of CM in the area of action of ADm and the area adjacent to it. The essence of this method of formation of flight routes of CM is as follows (Fig. 2) [6].

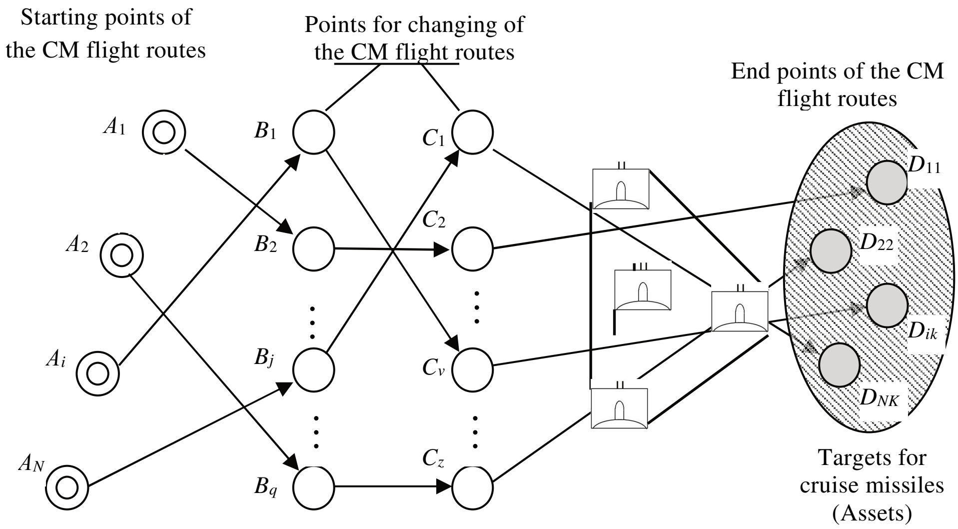

Fig. 2 Scheme of formation of cruise missiles flight routes (option)

Formation of CM flight routes is carried out for the probable directions of their strikes that are shown in Fig. 1. Two or three rows of possible points for changing the CM flight routes to the assets are designated in the area of operations of ADm. In addition, the starting A_i ($i=\overline{1,\ N}$) and final D_{ik} ($k=\overline{1,\ K}$) points on the CM flight routes are assigned (N – the number of CM in the strike, K – the number of the targets for cruise missiles (assets)). The points for changing the CM flight routes are assigned by the random value of number λ, which is distributed according to the uniform law in the interval [0; 1]. The following rule is used: to change the flight route of every i CM, point B_j is assigned if the condition is met

$$\frac{j-1}{q} < \lambda \le \frac{j}{q},\ j=\overline{1,\ q} \tag{1}$$

The points for changing the CM flight routes C_υ ($\upsilon=\overline{1,Z}$) are assigned similarly.

The proposed method makes it possible to form many CM flight routes in the area of action of ADm and thereby provide a more reliable assessment of their operation effectiveness.

The CM flight trajectories are formed in the local rectangular coordinate system, the beginning of which coincides with the location of the AD control point. In the same coordinate system, the locations of the positions of ADm and the assets are specified. According to the diagram in Fig. 2, for every i CM in the strike, the coordinates of the starting and ending points on the flight routes and the coordinates of the points of changing the flight routes are determined. The coordinates of the final points of the trajectories coincide with the coordinates of the location of the assets. The flight path parameters of CM are determined by interpolation steps, tabulated and used in simulations upon request. The initial simulation time corresponds to the initial coordinates of the points on the CM flight paths and is equal to zero.

When modeling the process of detecting CM by ADm, it is necessary to take into account the topography of the area in detail.

When modeling the flight trajectories of CM, the height at any point is determined as the sum of the terrain unevenness height and the cruise missile flight altitude. In this way, going around uneven terrain at a given height is simulated. To take into account the terrain of the area, it is advisable to use digital maps of the area.

The generalized structure of the AD formation is considered as consisting of formation command post (FCP) (combat command and control center, detection radar), several anti-aircraft missile battalions (combat command and control center, detection radar, several anti-aircraft missile batteries). The combat command and control centers of the FCP and the AAMb are connected to a network of detection radars.

In the case of simulation modeling, the execution of the following stages of the process of combat work on ADm is displayed [4-6]:

- detection of a CM by the radar of the FCP or a networked radar,
- bringing AAMb into combat readiness (as needed),
- distribution of CM between AAMb and issuing them with targeting,
- detection of CM by radars of AAMb, distribution of CM between AAMbt and issuing them with target designation,
- detection of CM by missile guidance systems (MGS) of AAMb and capturing them for escort,
- preparation of anti-aircraft guided missiles (AAGM) for launch,
- aiming AAGM and hitting the target.

In the simulation model, the stages of combat work can be taken into account in more details according to specific samples of ADm.

The logical sequence of modeling the stages of the combat work process in each implementation ($r=\overline{1,R}$) is as follows.

Formation of the flight trajectory of each i CM ($i=\overline{1,N}$) for strikes on assets protected by ADm is carried out in each implementation of simulation modeling.

The detection of each CM by the radar means of the AD formation is simulated. When a CM is detected by the radar of the FCP, the corresponding AAMb is brought into readiness for combat work and target designation is issued.

CM also can be detected by networked radar outside the battle order of the AD formation. Therefore, bringing the AAMb to combat readiness and issuing targeting can be carried out from the combat command and control center of the FCP based on the information from the networked radar.

When the radar of the command post of any AAMb detects a CM, target designation is issued to the corresponding AAMbt to capture the CM for escort by the MGS. If the cruise missile is detected by the MGS of the AAMbt, it is also simulated to capture it for escort.

In the case when the i CM was not detected, the transition is made to the simulation of the process of detecting the next CM from the strike. When a CM is detected, the preparation of AAGM for launch, the launch of AAGM and the destruction of the CM are simulated.

After the simulation of the CM defeating process by ADm, the transition to the simulation of the next implementation is carried out. The calculations of the effectiveness of assets protection by ADm from CM strikes are carried out, based on the results obtained from R implementations of the simulation.

Thus, in the model, calculations are organized according to a given number of realizations (R). Simulation of the process of defeating a CM by ADm is carried out sequentially for each CM. Limitations that affect the functioning of ADm must be taken into account when modeling. The main ones include restrictions on airspace survey sectors of radar means, sectors of responsibility of the AAMb, parameters of the damage zone of AAMbt, number of AAMbt channels, and ammunition of AAGM.

The structural diagram of the methodology of evaluating the effectiveness of assets protection by air defense means from cruise missiles strikes is shown in Fig. 3.

The FCP radar station, the networked radars, the radars of the AAMb, and the MGS of the AAMbt carry out the detection of the CM. When modeling the process of detecting CM by radar means, the approaches given in [12, 13] were used. The function of the detection range distribution of a CM by radar means was taken as the initial data in the simulation, taking into account the obstacle situation and the CM flight altitude $P_{\text{det}} = F(D_{\text{det}})$, which is usually determined based on the results of range tests. It is advisable to set the function $P_{\text{det}} = F(D_{\text{det}})$ in a table. Simulation of the fact (event) of the detection of a CM by separate radar means is carried out at each period of airspace inspection (T_{ins}). To do this, the location of the CM is determined based on the current flight time, its presence in the inspection sector is checked, and the range of the CM to radar means is calculated (d).

Next, the possible closing angle is determined, which depends on the height of the barriers (h_{bar}) and the distance of the barriers from the location of the radar means (d_{bar}). The closing angle is determined by the formula

$$\varepsilon = \arctan \frac{h_{\text{bar}}}{d_{\text{bar}}} \tag{2}$$

The potential capabilities of radar means in terms of target detection range (D_{det}) at low altitudes with normal atmospheric refraction of radio waves and flat terrain is determined by the formula [4-6]

$$D_{\text{det}} = 4.12\left(\sqrt{H_{\text{tar}}} + \sqrt{h_{\text{ant}}}\right) \tag{3}$$

where H_{tar} – the flight of target altitude, [m],

h_{ant} – the elevation height of the electrical center of the antenna relative to the surface of the radar station location, [m].

The detection range of radar means, taking into account closing angles (d_{det}) is determined by the formula [4]

$$d_{\text{det}} = \frac{1}{\cos\varepsilon}\left[d_{\text{bar}} + \left(D_{\text{det}} - d_{\text{bar}}\right)\left(1 - \frac{d_{\text{bar}}\tan\varepsilon}{H_{\text{tar}}}\right)\right] \tag{4}$$

When $d \le d_{\text{det}}$ from table $P_{\text{det}} = F(D_{\text{det}})$ based on the value of the current range d, P_{det}, probability of detecting CM is selected which is compared with a random number ξ, which is uniformly distributed in the interval [0, 1]. A cruise missile detection event is considered to occur when $P_{\text{det}} \ge \xi$.

In this way, the detection of a CM is simulated by all radars of AD formation and networked radars, which are designed to perform this task. At the same time, it is considered that the CM was detected by radar with the minimal current detection time.

The current flight time of i CM when simulating the process of its detection by a specific radar mean is determined taking into account the period of airspace survey T_{syr} of this radar mean as follows:

$$t_i = t_i + T_{\text{syr}} \tag{5}$$

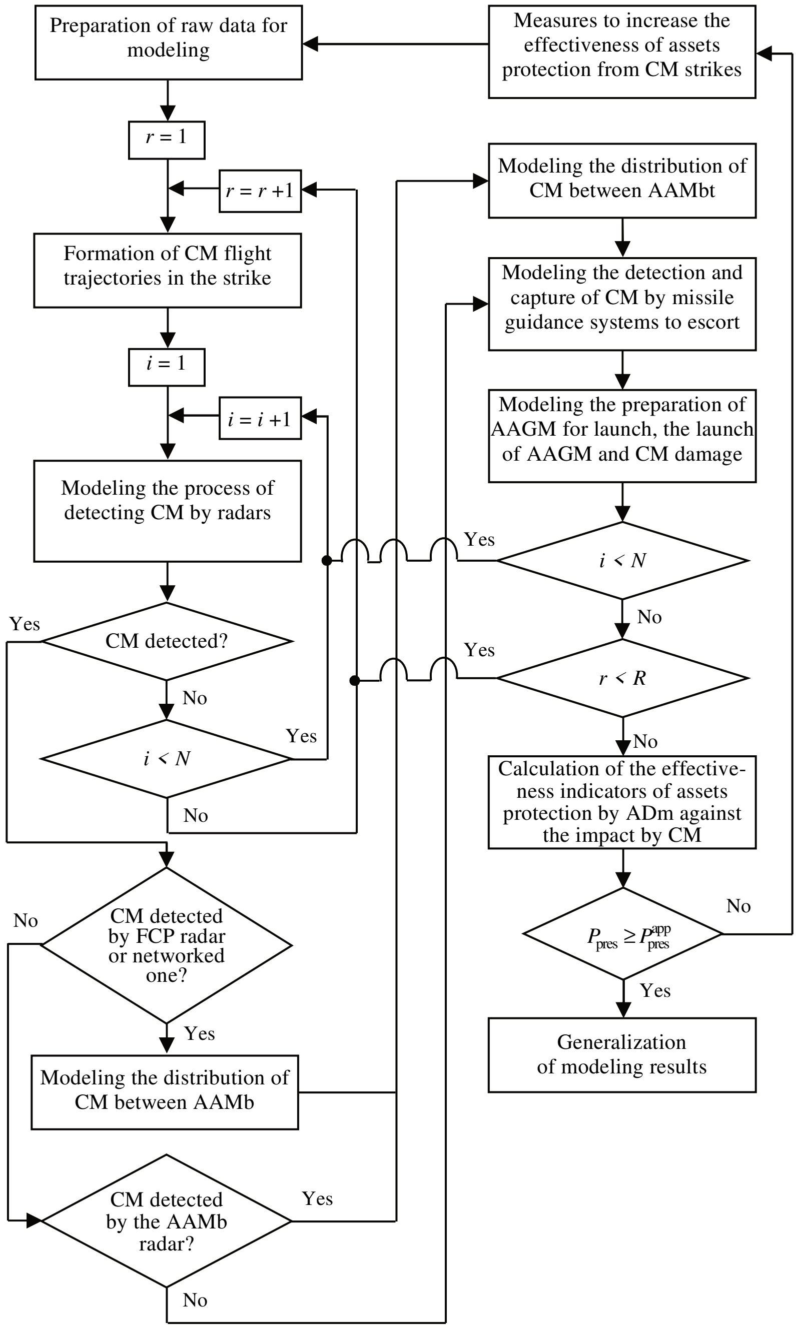

Fig. 3 Structural diagram of evaluation methodology of effectiveness of assets protection by air defense means from cruise missiles strikes

When modeling the distribution of target at the FCP combat command and control center, the following is taken into account: the location of the CM in the sector of responsibility of the AAMb, the course parameter and the flight time of the CM to the AAMb. The current time of solving the problem of target allocation and the assignment of the corresponding AAMb to attack *i* CM is calculated as follows?

$$t_{\text{all}i}^{*} = t_{\text{det}i} + T_{\text{all}}^{*} \tag{6}$$

where $t_{\text{det}i}$ – the current detection time of *i* cruise missile by the radar station of the AD formation command post or the networked radar,

T_{all}^{*} – the time spent on targeting and setting the task of the AAMb.

When modeling the target distribution at the AAMb combat command and control center, the following is taken into account: the long and short boundaries of the target distribution; sectors of responsibility of AAMbt; availability of free target channels of AAMbt, course parameter and flight time of CM.

The current time for solving the task of target allocation and issuing target indication for the AAMbt is determined as follows:

$$t_{\text{all}i} = t_{\text{all}i}^{*} + T_{\text{all}} \text{ or } \quad t_{\text{all}i} = t_{\text{detb}i} + T_{\text{all}} \tag{7}$$

where T_{all} – the time spent on target allocation and issuance of target indication for the AAMbt,

$t_{\text{det}i}$ – the current time of detection the *i* CM by the radar station of AAMb command post.

The current flight time of the CM corresponds to the near boundary of the target allocation $t_{\text{nall}i}$. When $t_{\text{nall}i} > t_{\text{all}i}$ the CM is considered unfired.

Current detection time of *i* CM by a MGS based on targeting

$$t_{\text{targ}i}^{*} = t_{\text{all}i} + T_{\text{targ}} \tag{8}$$

where T_{targ} – the time spent on receiving targeting and detecting a CM.

The time of capturing a CM for escort by a MGS $t_{\text{esc}i}$ is considered to be distributed according to a normal law. Therefore, the current time of capture of the *i* CM for escort is calculated as follows:

$$t_{\text{esc}i} = t_{\text{targ}i} + \overline{T_{\text{esc}}} + \gamma\sigma_{T\text{esc}} \tag{9}$$

where $\overline{T_{\text{esc}}}$ – the mathematical expectation of the capture time of a CM for escort,

$\sigma_{T_{\text{esc}}}$ – the mean square deviation,

γ – the random variable that follows a standardized normal distribution [14].

In case the *i* CM was detected by MGS

$$t_{\text{esc}i} = t_{\text{detb}i} + \overline{T_{\text{esc}}} + \gamma\sigma_{T\text{esc}} \tag{10}$$

In the model the ranges to the far $d_{\text{flz}i}$ and near $d_{\text{nlz}i}$ launch zones of AAGM are determined. It is done taking into account the course parameter of the CM, the distances to the far and near limits of the AAMs' damage zone, the flight time of the AAGM to the far and near limits of the AAMs' damage zone, and the flight speed of the CM.

The current time of an AAGM readiness to be launched at the *i* CM

$$t_{\text{laun}i} = t_{\text{esc}i} + T_{\text{prep}} \tag{11}$$

where T_{prep} – the time of preparing the AAGM for launch.

The range to the cruise missile corresponds to this time $d_{\mathrm{laun}i}$. An anti-aircraft guided missile is launched when $d_{\mathrm{nlz}i} \le d_{\mathrm{laun}i} < d_{\mathrm{flz}i}$. When $d_{\mathrm{laun}i} < d_{\mathrm{nlz}i}$, the CM is considered unfired.

The initial data of the model specifies the conditional probability of a CM being struck by an AAGM' warhead P_{str}. When simulating the impact of a CM, the probability P_{str} is compared with a random number β, that is uniformly distributed in the interval [0; 1]. The cruise missile is considered struck when $P_{\mathrm{str}} \ge \beta$.

When two AAGMs are firing for CM, the conditional probability of it being hit $\left(P_{\mathrm{str}}^{*}\right)$ is determined by the formula

$$P_{\mathrm{str}}^{*} = 1-\left(1-P_{\mathrm{str}}\right)^{2} \tag{12}$$

Conditional probability P_{str}^{*} is also compared to a random number β. In case of striking the CM, the target channel of the AAMbt is released. If the CM was not struck, its re-firing is simulated. In this case, the time spent preparing the AAGM for launch is also taken into account as T_{prep}.

For each implementation of the simulation, the mathematical expectation of the number of destroyed CMs from the strike is determined $m_{\mathrm{des}r}$.

The mathematical expectation of the relative number of CMs that are destroyed by ADm during their strike (M_{des}) being reflected is determined by the formula

$$M_{\mathrm{des}} = \frac{\sum_{r} m_{\mathrm{des}r}}{RN},\ r=\overline{1,R} \tag{13}$$

The numbers of CM for the damaging of the k asset (N_{k}) are given in the initial data of the model. According to the simulation results, the mathematical expectation of the number of CMs that will be hit from the composition of those designed to hit each asset is determined ($m_{\mathrm{des}k}$).

Probability of assets preservation from CM strikes P_{pres} is determined by the formula

$$P_{\mathrm{pres}} = \sum_{k} \omega_k \left(1-P_{1k}\right)^{N_k - m_{\mathrm{des}k}};\ \ k=\overline{1,K};\ \ \sum_{k}\omega_k = 1;\ \ \sum_{k} N_k = N;\ \ \sum_{k} m_{\mathrm{des}k} = m_{\mathrm{des}} \tag{14}$$

where ω_k – the importance coefficient of the k asset,

P_{1k} – the probability of damaging the k asset by one CM.

To determine the importance coefficients of assets ω_k, expert evaluation methods are used, in particular, the ranking method [15, 16].

The probability P_{1k} for the particular asset is determined by the formula [17]

$$P_{1k} = 1-\exp\left(-\frac{\rho^2 R_{\mathrm{dz}k}^2}{B_{\mathrm{dev}}^2}\right) \tag{15}$$

where $\rho = 0.4769$,

$R_{\mathrm{dz}k}$ – the damage zone radius of the k asset by a cruise missile's warhead (specified in the initial data),

$B_{\mathrm{dev}} = 0.5(B_{\mathrm{r}} + B_{\mathrm{1d}})$, B_{r}, B_{1d} – the average deviations of the fall points of CM from the scattering center in the range and lateral direction are given respectively.

When the obtained probability of assets preservation P_{pres} is less than appointed $P_{\mathrm{pres}}^{\mathrm{app}}$, it is necessary to carry out measures to improve the effectiveness of assets protection from CM strikes. For example, it is needed to adjust the battle order of AD units, to improve the process of combat work or strengthen the composition of ADm.

The accuracy of determining the indicators M_{des} and P_{pres} depends on the number of implementations R in the computational experiment. It follows from the analysis (13, 14) that the determining factor is the accuracy of the mathematical expectation of the CM number that are destroyed

$$m_{\text{des}} = \frac{\sum_{r} m_{\text{des}r}}{R}, \ r = \overline{1, R} \tag{16}$$

The necessity in modeling implementations is determined in the following sequence [12, 13].

They are given by the confidence interval ε and confidence probability α. Their meanings are usually taken as follows $\varepsilon = 0.05\text{-}0.10$, $\alpha = 0.90\text{-}0.95$.

The mean squared error is determined

$$\sigma_{\text{des}} = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\left(m_{\text{des}r} - m_{\text{des}}\right)^2} \tag{17}$$

The confidence interval is determined by the formula [18]

$$\varepsilon_{\text{M}} = \frac{\sigma_{\text{des}}}{\sqrt{R}} t_{\alpha} \tag{18}$$

where t_{α} – the tabular value of Student's distribution function.

When $\varepsilon_{\text{M}} > \varepsilon$, the simulation must be continued.

When developing the model, the stages of the combat work process and the sequence of individual operations at these stages are summarized for medium- and short-range AAMs. This makes it possible to apply the methodology with minor changes, which mainly concern the names of operations, and to evaluate the effectiveness of assets protection from CM strikes with AD formations armed with Patriot, NASAMS, IRIS-T SLM, S-300, Buk-M1 and other AAMs.

The methodological provisions given in the article take into account both the peculiarities of the usage of CM to strike the assets, and the peculiarities of the functioning of ADm when repelling the strike. This, first of all, concerns the formation of the CM flight trajectories in the model and the need of taking into account in detail the sequence of execution of the stages of combat work and operations on ADm when repelling CM strikes.

The application of a random change in the direction of CM flight, when modeling their trajectories in the area of ADm action and the area adjacent to it, is new in the developed methodological provisions. This makes it possible to take into account the uncertainty of the usage of CM to strike the assets. In addition, the modeling of the CM flight trajectories in the model is carried out taking into account the contour of the terrain, which is important when modeling the process of their detection by radar means of AD formation.

The detailed consideration in the model of the combat work stages and operations related to their execution allows objectively assessing the impact of these operations on the effectiveness of assets protection by ADm from CM strikes. This makes it possible to substantiate recommendations for improving the battle orders of AD formation units and the process of their combat work.

To evaluate the effectiveness of ADm work, the methodology uses an indicator – the probability of assets preservation from CM strikes. In comparison with the mathematical expectation of the number of destroyed CM, such an indicator better corresponds to the purpose of using ADm, that is, it is the most representative.

It is known [6] that the number of targets destroyed by ADm essentially depends on the strike density.

As an example, the application of the considered methodology for evaluating the influence of the density of CM strike on the probability of assets preservation from their strike is considered. It is assumed that the attack of CM of the X-101, X-555 types is carried out at an altitude of 100 m from the direction that corresponds to the sector of responsibility of one AAMb. It consists of three AAMbts, which have four target channels each. Flight speed of a CM V_{CM} = 300 m/s. Thirty cruise missiles hit five assets of varying importance.

The initial data and results of the calculation of the probability of hitting asset with one cruise missile are given in Tab. 1.

The initial data (regarding the possibilities of detecting a CM (the function of the distribution of the ranges of the detection of CM by radar means), the parameters of the sector of responsibility of the AAMb, the parameters of the damage zone of the CM by the AAMs and the time characteristics of the combat operation process) is generalized and given for the AD formation means, which are armed with medium-range AAMs. Conditional probability of hitting a CM by one AAGM $R_{\mathrm{h}} = 0.7$. Two AAGM are used to damage the CM. The number of implementations of the model $R = 1\,000$.

Tab. 1 Initial data and results of calculation of probability of hitting asset with one cruise missile

Asset number, k	Importance coefficient of asset, ω_k	Number of CM that are intended to strike asset, N_k	Ratio, $\frac{R_{\mathrm{dz}k}}{B_{\mathrm{dev}}}$	Probability of damaging asset by one CM, P_{1k}
1	0.11	3	0.90	0.169
2	0.21	7	1.00	0.204
3	0.35	9	1.15	0.260
4	0.15	5	1.30	0.320
5	0.18	6	0.80	0.136

The simulation was carried out for the strike density of CM Q = 1; 3; 5; 10; 15 CM/min. The given probability of assets preservation $P_{\mathrm{pres}}^{\mathrm{app}} = 0.7$.

The results of evaluating the mathematical expectations of the number of CMs destroyed by ADm depending on the density of the strike are given in Tab. 2.

The results of calculating the probability of assets preservation from cruise missiles strikes of different densities according to Eq. (14) are shown in Fig. 4.

It follows from the above results that the appointed probability of assets preservation $P_{\mathrm{pres}}^{\mathrm{app}} = 0.7$ in the given conditions is ensured at the strike density of the CM up to 3 CM/min. If the predicted strike density of the CM plaque is greater, then it is necessary to carry out measures to strengthen the protection of the most important assets first.

Tab. 2 Mathematical expectations of the number of Cruise Missiles destroyed by Air Defense Means

Strike density Q, [CM/min]	Strike duration of CM T, [min]	The entry interval of CM into the effect area of ADm, Δt, [s]	Mathematical expectation of CM number that are destroyed when object is protected by ADm, units.					Mathematical expectation of relative number of CM that are destroyed by ADm, M_{des}
			m_{des1}	m_{des2}	m_{des3}	m_{des4}	m_{des5}	
1	30	60	2.58	6.02	7.74	4.30	5.16	0.86
3	10	20	2.43	5.67	7.29	4.05	4.86	0.81
5	6	12	2.19	5.11	6.57	3.65	4.38	0.73
10	3	6	1.92	4.48	5.76	3.20	3.84	0.64
15	2	4	1.50	3.50	4.50	2.50	3.00	0.50

In the example under consideration, the mean squared error $\sigma_{des.} = 1.31$ CM. The confidence interval is calculated according to the Eq. (18), for $\alpha = 0.95$ and $R = 1\,000$, $t_\alpha = 1.96$, $\varepsilon_M = 0.081$, which indicates the sufficiency of implementations (runs) of the model in the computational experiment.

These dependencies (Fig. 4) when organizing the AD of assets by the military management authorities allows to reasonably determine measures to ensure the required level of their protection against cruise missile strikes. It determines the practical orientation of the methodology.

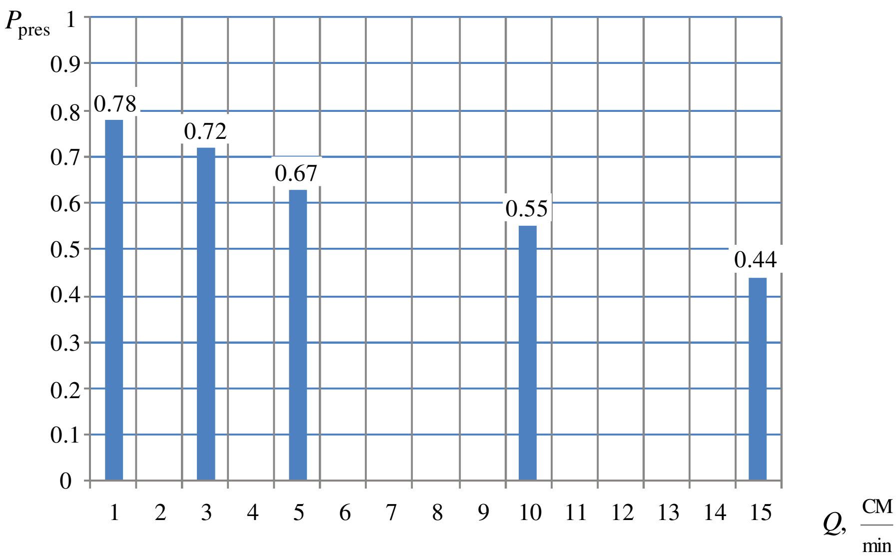

Fig. 4 Dependence of probability of assets preservation on density of cruise missiles strike

4 Conclusions

As an indicator for evaluating the effectiveness of assets protection from CM strikes, it is proposed to use the probability of their preservation. The probability of assets preservation is determined taking into account their importance.

A methodology has been developed for evaluating the effectiveness of assets protection from CM strikes by AD formation armed with medium- and short-range AAMs.

The battle order of AD units is built taking into account the probable directions of the CM strikes. It is proposed to determine these directions by the presence of optically visible landmarks for the functioning of the CM control system.

The basis of the methodology is a simulation model of the functioning process of the ADm when repelling the CM strike.

The feature of the simulation model is the formation of a large number of CM flight routes to the assets with a random change of directions, which leads to a more credible assessment of the ADm application effectiveness.

The model simulates in detail all stages and operations of combat work on the ADm by time. It gives a possibility to assess objectively the effectiveness of the ADm application when repelling CM strikes and develop recommendations for improving the unit battle orders of AD formation and the process of combat work.

It is advisable to use the developed methodology to determine the required composition of ADm to protect assets from the CM strikes of different densities with an appointed effectiveness.

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