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TUBE CHARGE WITH PROFILED SURFACE

Reviewer: František LUDVÍK

A b s t r a c t :

The paper describes the properties of the solid propellant tube charge being profiled on the outer surface and introduces the procedure of the main geometric characteristics determination. It is mainly aimed at the course of the burning surface variation of this type of the solid propellant charge in individual phases of burning.

1. Introduction

The solid propellant charge (SP) having the shape of the tube with the outer profiled surface has been utilised in the development of the modern solid propellant rocket motors (SPRM). The SP charge of the tube shape with the outer profiled surface has relatively big burning surface. This is often used in case of the booster rocket motors, which has to accelerate the rocket in very short time to the required flight velocity. Very frequently the leaving of the rocket launcher is observed, where it is required that the rocket will have a certain leaving velocity (muzzle velocity). For such purpose can be with the advantage used the mentioned shape of the SP charge. The longitudinal grooves on the outer charge surface increase the initial burning surface and in such a way secure the initial level of the SPRM thrust.

2. Determination of the Main Solid Propellant Charge Dimensions

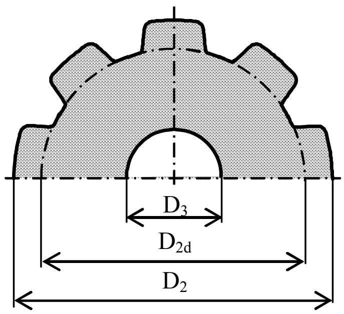

Fig. 1. Solid propellant charge cross-section

As it is evident from Fig. 1, the system of n – grooves is created on the outer surface of the SP charge. The shape of the groove and the main dimensions are evident in Fig. 2. The wall of the groove is inclined under the angle δ, the outer edge and the inner corner are rounded. Their radii are generally different.

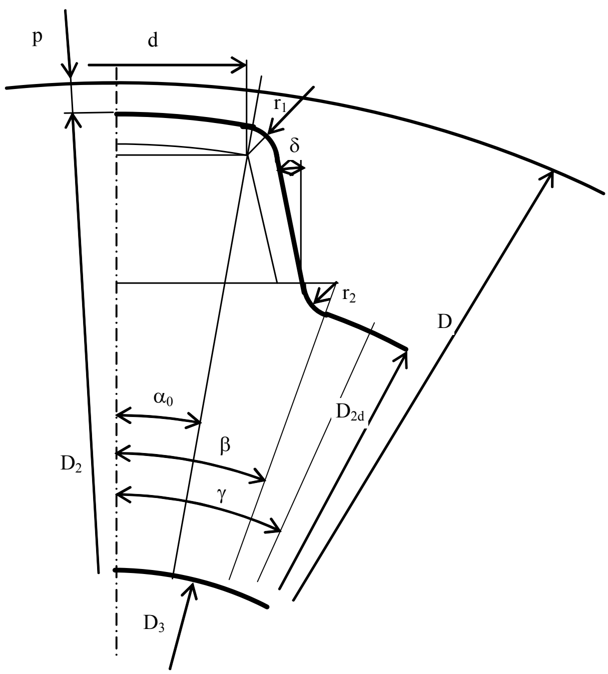

Fig. 2. The groove shape

For the groove shape construction the correct choice of the value *d* is important. At chosen value of *d* the determination of the angles α_0, β and γ is also important for the groove dimensions determination. The angle of the groove inclination δ from technologic reasons is usually greater as zero and its value is chosen till 10°. The angle of the grooves pitch is as follows:

$$\gamma = \frac{\pi}{n} .$$

The angle of the teeth width for the chosen value *d* is given by the relation

$$\alpha_0 = \sin^{-1} \frac{d}{D_2 - 2r_1} .$$

For the teeth bottom β can be used the procedure introduced in [1], i.e.:

$$x_A = \left(\frac{D_2}{2} - r_1 \right) \cos \alpha_0 ; \quad y_A = d .$$

$$y_0 = d + x_A \tan \delta + \frac{r_1 + r_2}{\cos \delta} ;$$

$$y_B = y_0 - x\, tg\delta .$$

$$R = \frac{D_{2d}}{2} + r_2 ; \tag{1}$$

$$x_B = \frac{y_0}{2} \sin(2\delta) + \sqrt{\left[\frac{y_0}{2} \sin(2\delta) \right]^2 - \cos^2(\delta)\left(y_0^2 - R^2\right)} ;$$

$$y_B = y_0 - x_B \tan \delta .$$

The angle of the teeth bottom is then

$$\beta = \cos^{-1} \frac{x_B}{R} .$$

The resulting value of the teeth bottom has to fulfil the condition $\beta \leq \gamma$.

The initial SP charge cross-section is given by the equation [2]:

$$A_{PH} = \frac{n}{4}\left(P_1 D^2 + P_2 D_2 + P_3 \right) - \frac{\pi}{4} D_3^2 , \tag{2}$$

where the respective substitutions are as follows

$$P_1 = \alpha_0 + c\sin(\beta - \alpha_0) + c^2(\gamma - \beta)$$

$$P_2 = 2(r_{1N} - r_{2N})\frac{\cos\alpha_0 - c\cos\beta}{\cos\delta} - 2(r_{N1}c - r_{2N})\sin(\beta - \alpha_0)\,; \tag{3}$$

$$P_3 = 4\left[r_{1N}r_{2N}\left(\frac{\cos\alpha_0 - \cos\beta}{\cos\delta} - \sin(\beta - \alpha_0)\right) + r_{1N}^2\left(\frac{\pi}{2} - \alpha_0 - \delta + \tan\delta - \frac{\cos\alpha_0}{\cos\delta}\right) - \right.$$
$$\left. - r_{2N}^2\left(\frac{\pi}{2} - \beta - \delta + \tan\delta - \frac{\cos\beta}{\cos\delta}\right)\right].$$

The surface burning on the whole outer part is given by the equation

$$S_0 = \Pi_0 K_L + 2A_{PH}\,, \tag{4}$$

where Π_0 is the initial SP charge burning perimeter.

The initial burning thickness is then as follows

$$e_0 = \frac{D_{2d} - D_3}{2}\,. \tag{5}$$

The most convenient SP charge dimensions can be determined during the procedure of the ballistic proposal being introduced e.g. in [1].

3. Variation of the Solid Propellant Charge According to the Time

The burning surface with respect to the time can generally be described by the differential equation:

$$\frac{dS}{dt} = \frac{dS}{de}\frac{de}{dt} = \frac{dS}{de}u\,. \tag{6}$$

For the charges burning on the lateral burning surface the introduced equation can be written as follows:

$$\frac{dS}{dt} = \frac{d(\Pi L_p)}{de}u + 2\frac{dA_{PH}}{de}u\,.$$

After some arrangement will hold:

$$\frac{dS}{dt}=\left(\frac{d\Pi}{de}L_P+\Pi\frac{dL_P}{de}+2\frac{dA_{PH}}{de}\right)u\ . \tag{7}$$

The instantaneous SP charge length is then:

$$L_P = L_{PH} - 2e\ , \tag{8}$$

and its derivative will then be

$$\frac{dL_P}{de}=-2\ . \tag{9}$$

The charge cross-section variation with respect to the time can be expressed by the equation:

$$\frac{dA_{PH}}{de}=-\Pi\ . \tag{10}$$

The burning surface variation after some arrangement is then as follows:

$$\frac{dS}{dt}=\left[\frac{d\Pi}{de}\left(L_P-2e\right)-4\Pi\right]u\ . \tag{11}$$

The equation (11) holds in the case of non protected fronts, i.e. for $L_P \neq$ cons. In case when L_P = cons. it holds more simple relation:

$$\frac{dS}{dt}=\frac{d(\Pi)}{de}L_P u\ . \tag{12}$$

In the further analyse the paper is related to the SP charges with protected (insulated) front, i.e. L_P = cons.

The determination of the burning perimeter of the mentioned SP charge is more complicated and it is necessary to express it in several phases (see Fig. 3).

The initial SP charge burning perimeter can be expressed by the equation [3]:

$$\begin{aligned}\Pi_0 = 2n\Bigg[&\frac{D_2}{2}\alpha_0+\frac{D_{2d}}{2}\left(\gamma-\beta\right)+\frac{D_2\cos\alpha_0-D_{2d}\cdot\cos\beta}{2\cos\delta}-\\&-\frac{r_1\cos\alpha_0+r_2\cos\beta}{\cos\delta}+r_1\left(\frac{\pi}{2}-\alpha_0-\delta+\tan\delta\right)+\\&+r_2\left(\frac{\pi}{2}-\beta-\delta+\tan\delta\right)\Bigg]+\pi D_3.\end{aligned} \tag{13}$$

1st phase - starts in the moment of the SP charge inflammation and is ended in the moment when the burning thickness *e* will reach the value r_1. The initial burning perimeter in the 1st phase is given by the equation:

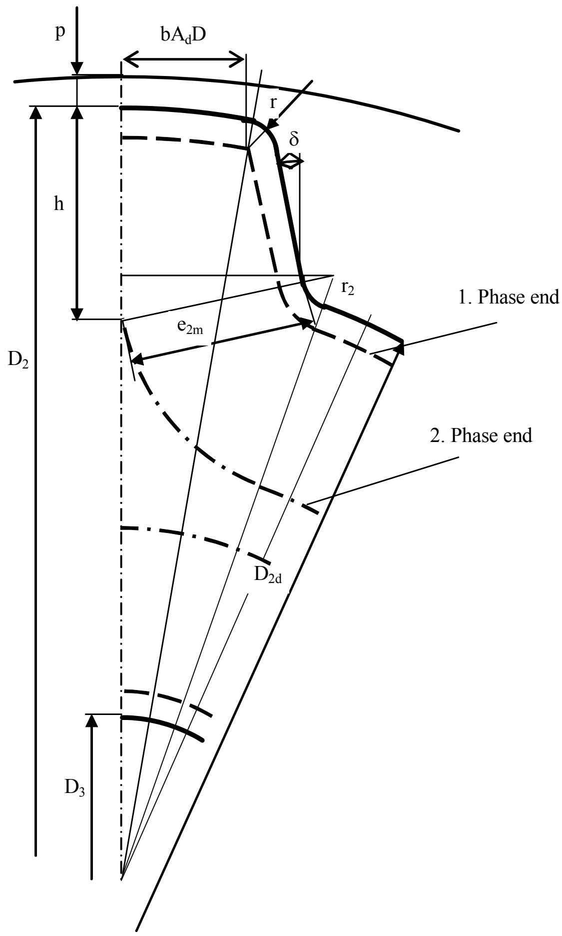

Fig. 3. Phases of the SP charge burning

$$\Pi_1 = 2n\left[\left(\frac{D_2}{2}-e\right)\alpha_0 + \left(D_{2d}-e\right)\left(\gamma-\beta\right) + \frac{D_2\cos\alpha_0 - D_{2d}\cos\beta}{\cos\delta} - \right.$$
$$-\frac{r_1\cos\alpha_0 + r_2\cos\beta}{\cos\delta} + \left(r_1-e\right)\left(\frac{\pi}{2}-\alpha_0-\delta+\tan\delta\right) + \tag{14}$$
$$\left. + \left(r_2+e\right)\left(\frac{\pi}{2}-\beta-\delta+\tan\delta\right)\right] + \pi\left(D_3+2e\right).$$

The derivative of the introduced equation according to the time is as follows:

$$\frac{d\Pi_1}{dt} = \left(-2n\gamma + 2\pi\right)u\ . \tag{15}$$

2nd phase - starts in the moment when the burnt thickness equals to $e = r_1$. The moment and the manner of the burning finishing in this phase depend on the relation of the teeth height h to the modified teeth width e_{2m} (Fig. 3). The teeth height can be expressed by the equation:

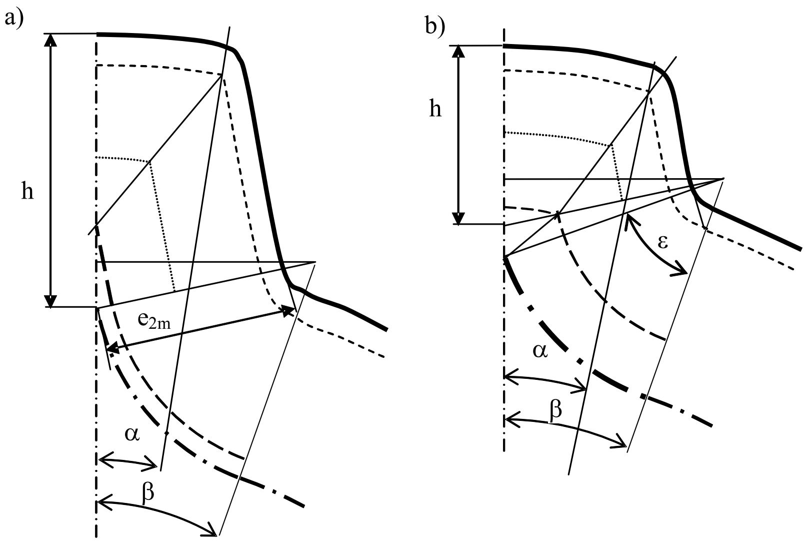

Fig. 4. Illustration of the SP charge 2nd phase of burning

a) for $h > e_{2m}$; b) for $h < e_{2m}$.

$$h = \frac{1}{2}\left[DA_D + \left(DA_D c + 2r_2\right)\left(\sin\beta\tan\delta - \cos\beta\right)\right], \tag{16}$$

The modified teeth thickness is given by the equation:

$$e_{2m} = \frac{1}{2}\left[\left(DA_D c + 2r_2\right)\frac{\sin\beta}{\cos\delta} - 2r_2\right]. \tag{17}$$

Comparing these two values follows then three possibilities, i.e.:

- When $h = e_{2m}$, the end of the 2nd phase is as marked in Fig. 3. But it is needed to conclude that such a case is exceptional only and that in the majority of cases is correct one of the following two cases.
- When $h > e_{2m}$, than in a certain moment of time such thickness is burnt when the angle $\alpha = 0$. From each teeth the triangle remainder then remains which burns in relatively very short time (Fig. 4. a).
- When $h < e_{2m}$, in certain moment of time the lateral teeth wall then disappears and during a certain time the teeth curve burns (Fig. 4. b)

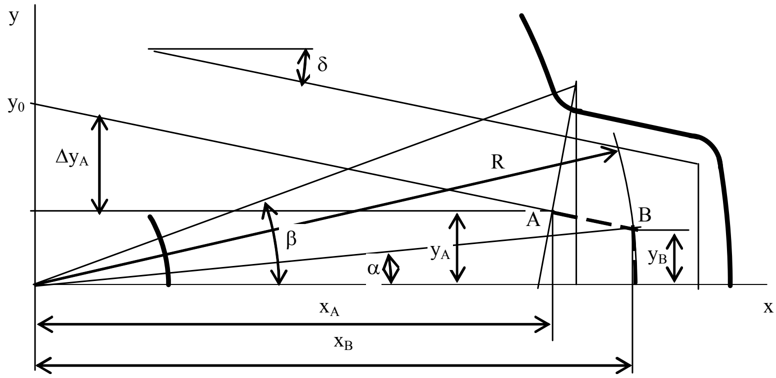

Fig. 5. Determination of the angle α

For the determination of the burning perimeter in this phase it is necessary to accurately determine the instantaneous value of the angle α. This can be solved by the procedure being given by the set of equations (1) for the points illustrated in Fig. 5.

The coordinates of the point A are as follows:

$$x_A = \left(\frac{D_{2d}}{2} + r_2\right)\cos\beta - \left(r_2 + e\right)\sin\delta\ ; \tag{18}$$

$$y_A = \left(\frac{D_{2d}}{2} + r_2\right)\sin\beta - (r_2 + e)\cos\delta$$

The value y_0 is then according to the Fig. 6:

$$y_0 = y_A + x_A \tan\delta \,. \tag{19}$$

The circle radius is as follows:

$$R = \frac{D_2}{2} - e \,. \tag{20}$$

X – coordinate of the point B can be obtained when substituting of the introduced values into the equation:

$$x_B = \frac{y_0}{2}\sin(2\delta) + \sqrt{\left[\frac{y_0}{2}\sin(2\delta)\right]^2 - \cos^2(\delta)\left(y_0^2 - R^2\right)} \;; \tag{21}$$

Y – coordinate of the point B will be obtained when solving the equation:

$$y_B = y_0 - x_B \tan\delta \,. \tag{22}$$

The angle α can be determined from the next equation:

$$\alpha = \sin^{-1}\frac{y_B}{R} \,. \tag{23}$$

Let us discuss the individual cases in more details.

1. the case - $h = e_{2m}$: during the whole time of the 2[nd] phase holds the following equation for the instantaneous burning perimeter:

$$\begin{aligned}\Pi_2 = 2n\Bigg[&\left(\frac{D_2}{2} - e\right)\alpha + \left(\frac{D_{2d}}{2} - e\right)(\gamma - \beta) + \frac{D_2\cos\alpha_0 - D_{2d}\cos\beta}{2\cos\delta} - \\ &-\frac{r_1\cos\alpha_0 + r_2\cos\beta}{\cos\delta} + (r_2 + e)\left(\frac{\pi}{2} - \beta - \delta + \tan\delta\right)\Bigg] + \pi(D_3 + 2e) \,,\end{aligned} \tag{24}$$

by its derivation we shell get:

$$\frac{d\Pi_2}{dt} = \left[2n\left(\frac{\pi}{2} - \alpha - \delta - \gamma + \tan\delta\right) + 2\pi\right]u \,. \tag{25}$$

The end of the 2nd phase is given by the condition $e \le e_{2m}$.

2. the case - $h > e_{2m}$ - the course of burning is given by the equations (18) till (24) as in the first case till the time moment when the angle α fulfil the condition $\alpha \le 0$. More convenient for the concrete solution is the condition $y_B \le 0$. From this time moment the burning perimeter for the remaining 2nd phase is given by the equation:

$$\Pi_{2a} = 2n\left\{\left[\left(\frac{D_{2d}}{2} + r_2\right)\frac{\sin\beta}{\cos\delta} - (r_2 + e)\right]\frac{1}{\tan\delta} + \left(\frac{D_{2d}}{2} - e\right)(\gamma - \beta) + \right.$$
$$\left. + (r_2 + e)\left(\frac{\pi}{2} - \beta - \delta\right)\right\} + \pi(D_3 + 2e). \tag{26}$$

Its derivative is as follows:

$$\frac{d\Pi_{2a}}{dt} = \left[n\left[\pi - 2\gamma - 2\delta - \frac{2}{\tan\delta}\right] + 2\pi\right]u\ . \tag{27}$$

The end of the 2nd phase is given by the same expression as in the previous case, i.e. the condition $e \le e_{2m}$.

3. case - $h < e_{2m}$ – the course of the burning is given by the equations (18) till (24) as in the 1st case till the moment when the teeth side wall is missed. In this moment the value x_B from the equation (21) is just equal to the value x_A from the equation (18). Further on it is necessary to solve the coordinates of the point B (see Fig. 7) till the time moment when the coordinate value y_B will reach the zero value.

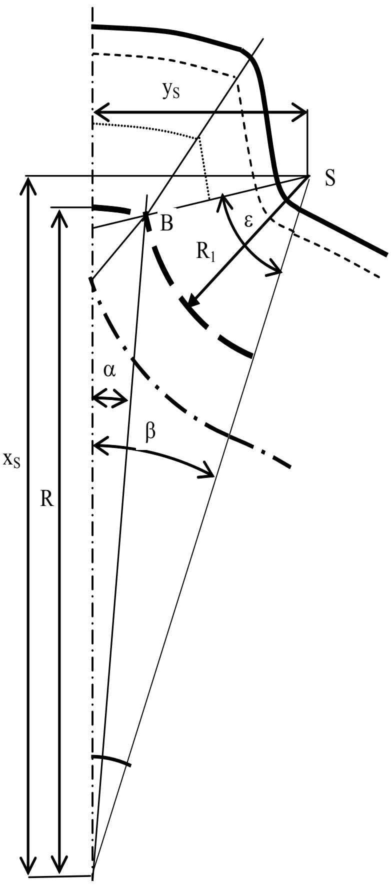

Fig. 6. Derivation of the conditions for the 2nd phase of burning

The burning perimeter of the remaining part of this phase depends on the coordinate of the point *B* (Fig. 6), which can be determined only as the coordinate of the intersection of the two circles. The 1st circle has the origin in the SP charge centre and the radius

$$R = \left(\frac{D_2}{2} - e \right), \tag{28}$$

The 2nd circle has the origin being given by the coordinates of the point *S* (x_S, y_S), where

$$x_S = \left(\frac{D_{2d}}{2} + r_2 \right) \cos \beta ; \tag{29}$$

$$y_S = \left(\frac{D_{2d}}{2} + r_2 \right) \sin \beta \ , \tag{30}$$

with the radius

$$R_1 = (r_2 + e) \ . \tag{31}$$

The coordinate x_B of the intersection point of two circles is as follows:

$$x_B = \sqrt{R^2 - y_B^2} = \sqrt{R_1^2 - (y_B - y_S)^2} + x_S \ .$$

The value of the coordinate y_B of the point B from the introduced equation is then as follows:

$$y_B = \frac{y_S \left(x_S^2 + y_S^2 + R^2 - R_1^2 \right) + x_S \sqrt{4R\left(x_S^2 + y_S^2 \right) - \left(x_2^2 + y_S^2 - R^2 - R_1^2 \right)^2}}{2\left(x_S^2 + y_S^2 \right)} . \tag{32}$$

The coordinate x_B is then:

$$x_B = \sqrt{R^2 - y_B^2} \tag{33}$$

The angle α is as follows:

$$\alpha = \sin^{-1} \frac{y_B}{R} \ , \tag{34}$$

and the angle ε (see Fig. 6) is as follows:

$$\varepsilon = \frac{\pi}{2} - \beta - \cos^{-1} \frac{(D_{2d} + 2r_2) \sin \beta - 2 y_B}{2(r_2 + e)} \ . \tag{35}$$

The burning perimeter for this part of the 2nd phase will then be:

$$\Pi_{2b} = 2n\left[\left(\frac{D_2}{2} - e \right) \alpha + (r_2 + e)\varepsilon + \left(\frac{D_{2d}}{2} - e \right)(\gamma - \beta) \right] + \pi (D_3 + 2e) \ . \tag{36}$$

The burning perimeter variation will be as follows:

$$\frac{d\Pi_{2b}}{dt} = \left[2n(\varepsilon - \alpha - \gamma + \beta) + 2\pi \right] u \tag{37}$$

In this case the 2nd phase is ended in the time moment when the value $y_B = 0$.

3rd phase – follows the termination the one introduced cases of the 2nd phase and is ended in the time moment when the whole burning thickness is given by the condition $e = e_0$.

The burning perimeter is composed from two parts only. The important is the determination of the angle ε, which is according to the Fig. 6 as follows:

$$\varepsilon = \frac{\pi}{2} - \beta - \cos^{-1}\frac{(D_{2d} + 2r_2)\sin\beta}{2(r_2 + e)} \tag{38}$$

The burning perimeter is then:

$$\Pi_3 = 2n\left[(r_2 + e)\varepsilon + \left(\frac{D_{2d}}{2} - e\right)(\gamma - \beta)\right] + \pi(D_3 + 2e). \tag{39}$$

The burning perimeter variation will then be:

$$\frac{d\Pi_3}{dt} = [2n(\varepsilon - \gamma + \beta) + 2\pi]u\ . \tag{40}$$

4. Solution examples

As the example for the discussed geometric shape of SP charge burning surface variation were chosen the following values: D_2 = *0.0811* m; D_{2d} = *0,073* m; D_3 = *0,0329* m; L_P = *0,6415* m; n = *10*; e_0 = *0,01* m; r_1 = *0,001* m; r_2 = *0,002* m; angles α_0 = *4,5°*, β = *9,33°*, γ = *18°*, δ = *6°*. The introduced values were determined from the ballistic proposal (design) and flows out from the allocation introduced in [1]. The resulting course of the burning surface variation is shown in Fig. 7. The course introduced in the 2nd phase of burning responds to 2nd phase condition - $h > e_{2m}$.

The SPRM thrust increase at the beginning of the RM can be increased by the grooves depth, but this has some limitation due to the groove thickness d limitation which depends on the choice of number of grooves and the angle of groove wall inclination δ. The following charge dimensions were chosen for this case: D_2 = *0,092* m; D_{2d} = *0,0782* m; D_3 = *0,015* m; the other dimensions remain the same. The angles α_0 and β were calculated from system of equations (1). The resulting course of the burning surface variation is shown in Fig. 8.

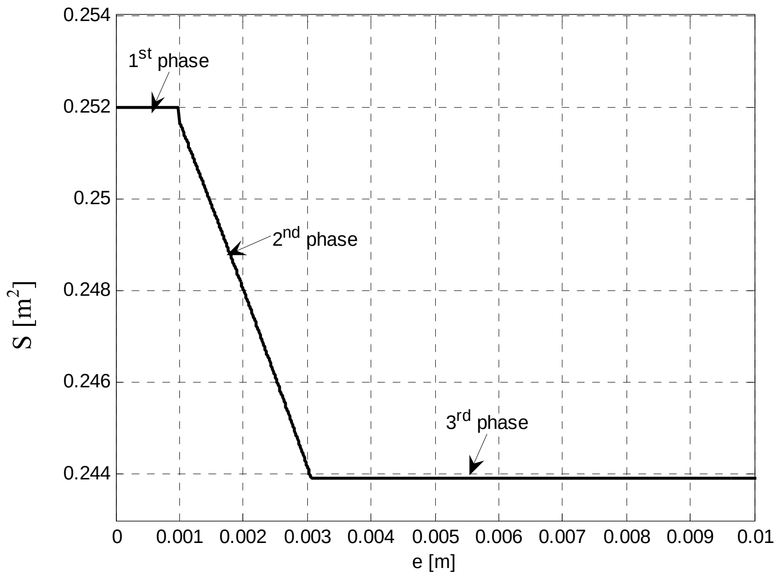

Fig. 8. Course of the solid propellant charge surface variation – 1st example

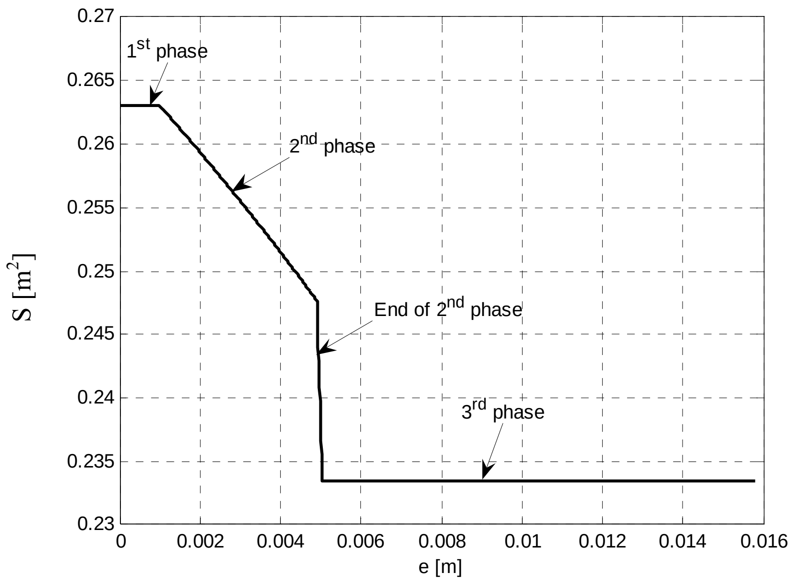

Fig. 8. Course of the solid propellant charge surface variation – 2nd example

5. Conclusion

The paper contributes to the military technologies development by the improvement of theory of the solid propellant rocket motor charge shapes. The process of solid propellant charge with longitudinal grooves on the outer surface solution updates the shape dimensions determination and brings the process of charge burning surface change in time. This process further provides the improvement of the main task of solid propellant rocket motor internal ballistics with introduced charge shape.

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