Anti-Ballistic Missile System
Selected Technical Principles of Function

I. Hamtil and M. Krátký[*]

Department of Air Defence Systems, University of Defence, Brno, Czech Republic

Abstract:

This paper deals with a probable defence scenario and the USA GMD/NMD Anti-Ballistic Missile system functioning during engagement of Middle Range Ballistic Missile (MRBM), or Intercontinental Range Ballistic Missile (IRBM) in midcourse, exospheric and high phase of the missile flight.

Keywords:

1. Introduction

It is expressed precisely in the source work [1], that the need of defence against ballistic missiles threat appeared with full urgency when during the 2nd World War, concretely on Friday September 8th 1944, when first German long-range ballistic rocket V-2 fell down to London. British, even before the end of war, had developed a defence concept against long-range ballistic missiles which is valid up to present time. However, technical problems had not made it possible to realize the project and it was set back. It seemed at that time, that a weapon was born, and there is no countermeasure against it. Only after air-defence missile systems were successfully developed in the second half of the fifties of the previous century, there was a new challenge to deal with ballistic missiles. Especially teams of the USA and USSR engineers had attempted to solve this challenge and their effort brought firstlings.

Nowadays USA, Russia, Israel, and recently NATO too, have been paying considerable attention to ballistic missiles defence (BMD). The contemporary USA and NATO BMD programs and projects [2-4] is coming out with requirement to create **integrated, non-nuclear, and multilayer BMD system** against all types of BM, it means against both short-range and long-range ballistic missiles. The system, which is able to engage them within all phases of flight (start/boost phase, ascent phase,

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[*] *Corresponding author: Department of Air Defence Systems, University of Defence in Brno, Kounicova 65, CZ-662 10 Brno, Czech Republic, phone: +420 973 442 336, E-mail: miroslav.kratky@unob.cz*

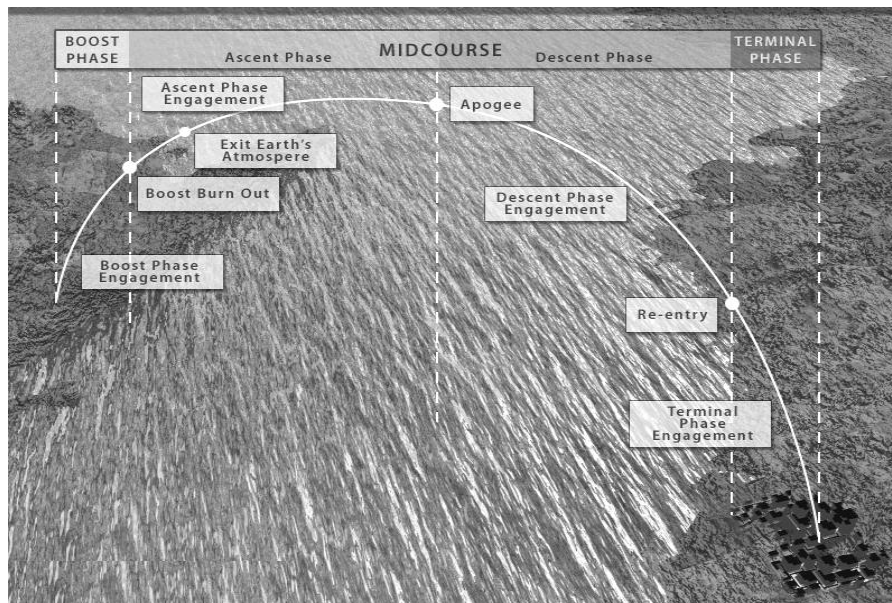midcourse/apogee phase, descent phase, re-entry phase, and terminal phase) is described by Fig. 1 [4].

Fig. 1 Phases of ballistic missile trajectory

A part of this system should be the Ground-based Midcourse Defence (GMD) which is built within the USA National Missile Defence (NMD) program. The role of the NMD ought to be the US territory defence against an intentional, but limited attack of several tens IRBM equipped by nuclear, chemical, or biological warheads, fired from a territory of state which has this type of weaponry at disposal, or non-intentional attack from Russia and China caused by a mischance, technical error, or non-authorized launch.

The attacking missiles have to be destroyed within their middle and high, exospheric phase of flight by kinetic energy – after impacted by so called **anti-rocket interceptor**. The range should be from 1000 to 5000 km, altitude from 130 to 1000 km.

The next NMD components are **sensors** [5, 6], both in the space (Satellites' System Defence Support Program / DSP; Space Based Infra-Red System / SBIRS), and on the surface (Upgraded Early Warning Radars / UEWR; X Band Radar / XBR; Sea Base Radar / SBR), stationary **launchers** and **anti-rockets**. The GMD interceptor's carriers are not fully developed yet. The BMD bases intended to be established in the Czech Republic and Poland should be the parts of this system, too. The whole system mentioned is illustrated by Fig. 2 [4].
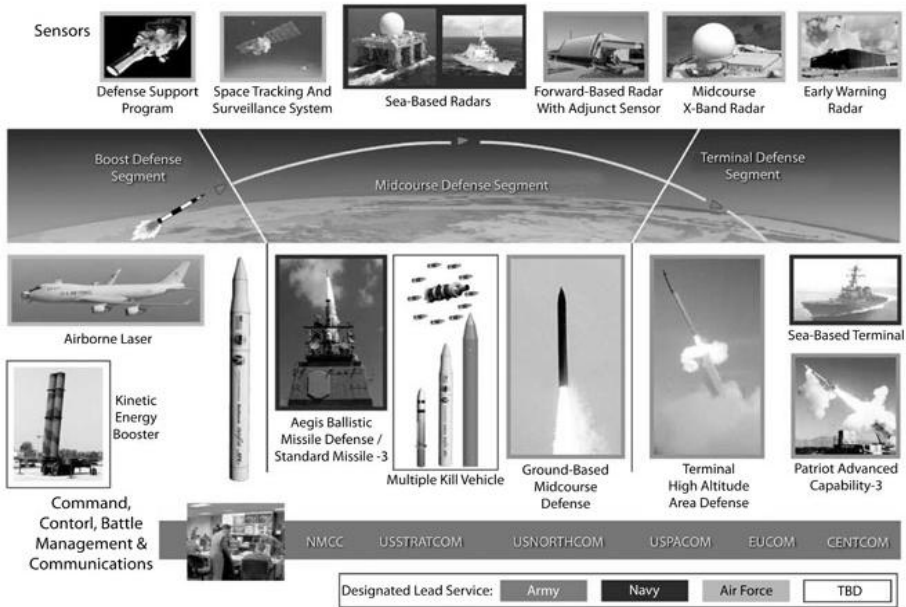
Fig. 2 Integrated Ballistic Missile System

2. GMD Structure and Principle of Operation

A presumable BM attack scenario and GMD principles of operation against middle-range (MRBM), or long-range (IRBM) ballistic missile within the middle, high and exospheric (*midcourse*) phase of BM flight will be described now.

A launch of ballistic missile is detected by DSP, or SBIRS satellites. They are tracking the BM and transmitting information about its trajectory, and a presumable type to the operational and command centre. Position of BM is determined via the ***triangular method*** when tracked by two satellites as minimum. On the basis of these data the relevant early warning radars (Upgraded Early Warning Radar / UEWR) are activated. These radars are tracing the BM and gathering necessary data for more accurate location and tracking.

To improve information about BM position and its probable trajectory, the optimally placed radars XBR or SBR (eventually AN/SPY-1 naval AEGIS system radar) take over the BM tracing. According to this measured data, the BM (or better to say warhead/re-entry vehicle) position, flight plane and next trajectory parameters are determined (the probable BM target position, trajectory apogee, time of flight). These precise radars should identify the true warhead (re-entry vehicle) among dummy targets.

The GMD anti-rocket (interceptor) is launched either according to the information from UEWR, or after the attacking BM is tracked by XBR (SBR, AN/SPY-1). The information gathered by radars is transformed to the guidance commands and sent to the interceptor board for its control system. ***The anti-rocket is guided into encounter (collision) course of the BM (warhead, re-entry vehicle) in the flight plane of BM***. After interceptor reaches the required speed and location (at an

altitude more than 130 km) an Exospheric Kill-Vehicle (EKV) separates from the anti-rocket carrier. EKV orients its sensor subsystem, points itself to the appointed target in accordance with command from radars, guides itself to the target and destroys it using kinetic interaction (direct, high-speed collision). There are plans to use "Multiple Kill Vehicle" (MKV) instead of EKV to engage more BMs/warheads in the future.

In the case the anti-rocket does not approach the BM in its flight plane, it means in traversing directions, the next factors are entering guidance process (time, or velocity). Thanks to it, the engagement probability is significantly declining.

3.   Determination of the BM Movement Parameters by XBR

It follows from mentioned above that the active phase of defence against middle-range or long-range BM starts with its detection, location and tracking by XBR (SBR, AN/SPY-1). After radar have eliminated decoys and aimed itself to the real (combat) warheads, it continuously determinates parameters of location and movement of warhead – see Figure 3.:

   - the distance between warhead (BM re-entry vehicle) and radar (figure r_i),

   - the target (BM re-entry vehicle) radial velocity toward radar position (figure $\dot{r}_i$),

   - the azimuth of radius vector (axis of sight) $\dot{r}_i$ of warhead (figure b_i),

   - the elevation of radius vector (axis of sight) $\dot{r}_i$ of warhead (figure e_i).

The first task which is performed by XBR radar on the basis of the attacking BM (warhead, re-entry vehicle) tracing and tracking is ***determination of location and the plane direction of the BM flight in reference to radar position***.

Primary situation and geometric relations between the BM flight plane direction and radar position are depicted in Fig. 3.

This is known from solid geometry, and from Fig. 3 it is obvious that the BM flight plane direction can be defined using two angles:

   - angle d_{0i} between connection line "centre of the Earth" O_z and radar position R, and its projection O_zP to BM flight plane; for this angle proceeds [7-9]:

$$d_{0i} = \arctan\left[\frac{r_i \cos e_i}{R_z + r_i \sin e_i} \cdot \cos(b_{0i} - b_i)\right] \tag{1}$$

   - angle b_{0i} which determines the orientation of plane defined by points O_z, P and radar position R towards the reference direction (N – North); this plane is perpendicular to BM flight plane; it is applied to b_{0i} [7-9]:

$$b_{0i} = \arctan\left(\frac{\tan d_i \cos b_i - \tan d_{i+1} \cos b_{i+1}}{\tan d_{i+1} \sin b_{i+1} - \tan d_i \sin b_i}\right). \tag{2}$$
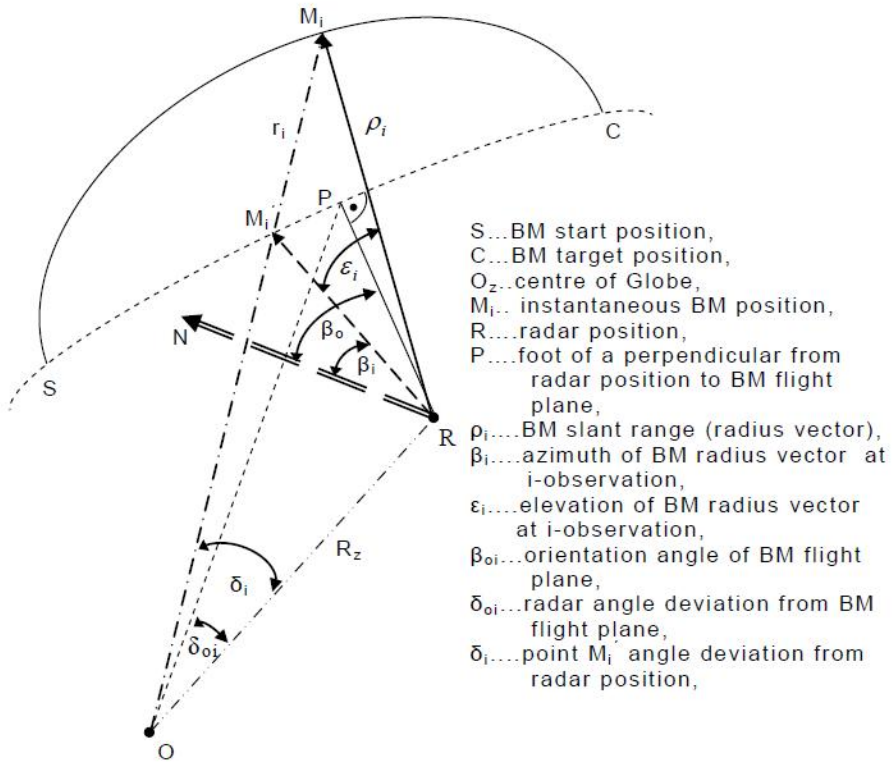
*Fig. 3 Geometric relations between BM plane direction and radar position
(index "i" means ith measurement/observation in time t$_i$)*

Then, the next task is the BM trajectory basic calculation (BM start and target position, height of apogee, time of flight) within the given BM flight plane. This is made by using so-called ecliptic theory, respectively Keplerian laws – i.e. considering flight in free space.

First of all (see Fig. 4) angles c_{0i} and c_i, are set up. These angles define BM position and its apogee towards the flow line between centre of the Earth and point P. The angles will be used during the BM ecliptic trajectory parameters determination. According to [7, 8] applies to c_i:

$$c_i = p \pm \arctan\left[\sin d_{0i}\tan\left(b_{0i} - b_i\right)\right]. \tag{3}$$

Note: Negative sign "–" in formula (3) appertains to the case when BM movement is done in accordance with "Fig. 4 situation"; i.e. from left to right side. Sign "+" refers to an event when BM is moving in the opposite direction.
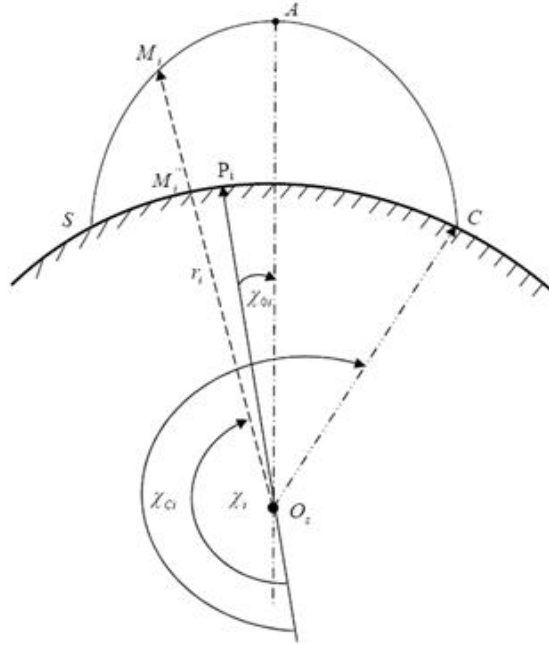
Fig. 4 Geometrical relationships for the BM ecliptic trajectory determination

It follows from so called "ecliptic theory" of ballistic missiles movement in the free space (it means outside the atmosphere) and Keplerian laws, that for quantity r_i the BM ecliptic trajectory (see Fig. 4) applies formula [7-9]:

$$r_i = \frac{S_v^2}{K[1 + e\cos(c_i - c_{0i})]}, \; S_v = r_i^2 \, \dot{c}_i, \; K = GM_z, \tag{4}$$

where

e ....ellipse eccentricity,

S_v ....BM's areal velocity [m²·s⁻¹],

$G = 6.67 \times 10^{-11}$ [kg⁻¹·m³·s⁻²].............gravitational constant,

$M_z = 5.95 \times 10^{24}$ [kg].........................mass of the Earth.

The procedure of determination S_v and e is described in detail in literature [2]. They are acquired from the BM position (r_i, b_i, e_i) and movement ($\dot{c}_i$) parameters multiple measuring, using XBR. Formulas (1) to (4) are used for the calculation.

According to [2] S_v (*areal velocity*) of BM elliptical trajectory is obtained as a standard solution of quadratic equation

$$AS_n^2 + BS_v + C = 0, \tag{5}$$

where

$$A = R_z \cos d_0 \left(\frac{\cos c_{i+1}}{r_i} - \frac{\cos c_i}{r_{i+1}} \right) + \cos(c_i - c_{i+1}) - \frac{r_i}{r_{i+1}},$$

$$B = \mathbf{\&} r_i \sin(c_{i+1} - c_i),$$

$$C = K(r_i + R_z \cos c_i \cos d_0)[1 - \cos(c_i - c_{i+1})],$$

where only real and positive root makes sense.

The next step is parameter *e* calculation, i.e. *BM elliptical trajectory eccentricity.* Using the XBR parameters, there is in [2] derived formula

$$e^2 = \frac{1}{\sin^2(c_{i+1} - c_i)} \cdot \left[\left(\frac{S_v^2}{K r_i} - 1 \right)^2 + \left(\frac{S_v^2}{K r_{i+1}} - 1 \right)^2 - 2 \cdot \left(\frac{S_v^2}{K r_i} - 1 \right) \left(\frac{S_v^2}{K r_{i+1}} - 1 \right) \cos(c_{i+1} - c_i) \right]. \quad (6)$$

The calculated parameters S_v and *e* are then utilized for the assessment of angle c_0, the size of main half-axe of elliptical trajectory *a*, and altitude of apogee h_A above Earth's surface. Refer to [1] hence it follows:

$$c_{0i} = \arctan \left[\frac{(S_v^2 - K r_i) r_{i+1} \cos c_{i+1} - (S_v^2 - K r_{i+1}) r_i \cos c_i}{(S_v^2 - K r_{i+1}) r_i \sin c_i - (S_v^2 - K r_i) r_{i+1} \sin c_{i+1}} \right], \quad (7)$$

$$a = \frac{S_v^2}{K(1 - e^2)}, \quad (8)$$

$$h_A = a(1 + e) - R_z, \quad (9)$$

The instantaneous value of BM height of flight above Earth's surface at i^{th} measurement can be determined in accordance with formula [2]:

$$h_i = \frac{a(1 - e^2)}{1 + e \cos(c_i - c_0)} - R_z, \quad (10)$$

Presumed that BM's height of flight equals zero ($h_i = 0$) and derived the formula (10) the value of angle $c_i = c_C$, $c_C \in (p \div 2p)$ is obtained, which define target position in the plane flight (see Fig. 4):

$$c_C = c_0 + \arccos \left[\frac{a(1 - e^2) - R_z}{e R_z} \right]. \quad (11)$$

Thus, the place of BM's start position is situated within BM's flight plane and it is symmetric to the target position. The axe of symmetry is the connecting line between $O_z A$ – see Fig. 4. The appropriate angle $c_S \in (0 \div p)$ is assessed by formula:

$$c_S = 2(p + c_0) - c_C, \quad (12)$$

It is more practical to specify the target position using geographical coordinates, i.e. geographical latitude j_Z and longitude l_Z. The tracking radar position (XBR, SBR, AN/SPY-1) is usually localized by these coordinates too.

Since heretofore the position and movement characteristics of tracked BM refer to XBR position, it is not difficult to determine geographical coordinates of BM start and target/impact place.
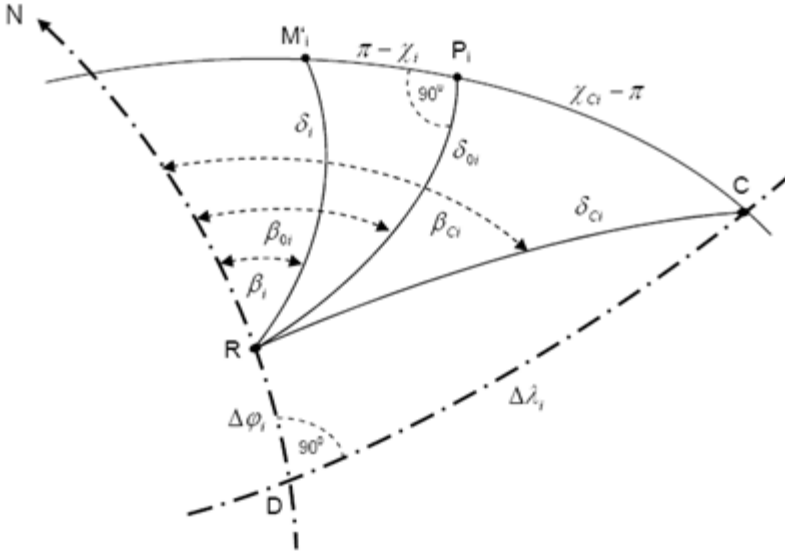
Fig. 5 Determination of BM's position

Rules of spherical trigonometry can be used again (see Fig. 5) and it is possible to determine magnitudes b_{Ci} and d_{Ci} which characterize target position towards radar position. This applies to rectangular triangle CPR [7, 8]:

$$b_{Ci} = b_{0i} + \arctan\left[\frac{\tan(c_{Ci} - p)}{\sin d_{0i}}\right], d_{Ci} = \arctan\left[\frac{\cos(b_{Ci} - b_{0i})}{\tan d_{0i}}\right], \tag{13}$$

There is the target deviation calculation from radar position (magnitudes Δj_i and Δl_i) from rectangular spherical triangle CDR. It applies for this triangle:

1. $b_{0i} \geq p/2$:

$$\tan(\Delta j_i) = \tan d_{Ci} \cdot \cos(p - b_{Ci}) \Rightarrow \Delta j_i = \arctan(-\tan d_{Ci} \cdot \cos b_{Ci}), \tag{14}$$

$$\tan(\Delta l_i) = \tan d_{Ci} \cdot \sin(p - b_{Ci}) \Rightarrow \Delta l_i = \arctan(\tan d_{Ci} \cdot \sin b_{Ci}). \tag{15}$$

2. $b_{0i} < p/2$:

$$\tan(\Delta j_i) = \tan d_{Ci} \cdot \sin(p/2 - b_{Ci}) \Rightarrow \Delta j_i = \arctan(\tan d_{Ci} \cdot \cos b_{Ci}), \tag{16}$$

$$\tan(\Delta l_i) = \tan d_{Ci} \cdot \cos(p/2 - b_{Ci}) \Rightarrow \Delta l_i = \arctan(\tan d_{Ci} \cdot \sin b_{Ci}). \tag{17}$$

And for standard geographic coordinates:

$$j_{ZC} = j_{ZR} + \Delta j_i, \ l_{ZC} = l_{ZR} + \Delta l_i. \tag{18}$$

The geographic coordinates of BM start position are calculated analogically, only instead of angle c_{Ci}, the value c_{Si} is used – according to formula (12).

The time of BM flight from position M_i, where it is located in time t_i, up to the BM target, can be determined by solution of differential equation (19) which has been obtained by formula (4) modification:

$$\dot{c}_i(t) = \frac{S_v}{r_i^2(t)}, \quad \text{where} \quad r_i(t) = \frac{S_v^2}{K + Ke\cos[c_i(t) - c_{0i}]}. \tag{19}$$

Differential equation (19) is solved by acceptance the initial and edge conditions:

$$\text{if} \quad t_i = t_1 \quad \text{then} \quad c_i = c_1 \quad \text{and} \quad \text{if} \quad t_i = t_C \quad \text{then} \quad c_i = c_C.$$

Then the differential equation (19) is solved, when angle c gets the value $c_i = c_C$. It comes if $t_i = t_C$. In such a way, the information how long BM flies from M_i position to the target is got.

4. Requirements for Configuration of GMD ABM System

The GMD system configuration is determined firstly by mutual radar XBR and anti-rocket position emplacement, secondly by both the GMD system components situating towards presumptive BM planes of flights.

It is a prerequisite from the viewpoint of the operation principle GMD systems which affects against ballistic missiles in exospheric phase of flight that an anti-rocket *launcher position should be as close as possible to the presumed flight planes of attacked ballistic missiles. This allows anti-rockets to get the collision encounter course in the BM flight plane with minimum delay and manoeuvre required*.

Another situation is for the aiming, tracking and guidance XBR (SBR, AN/SPY-1) radar position choice. Considering a complexity of mathematical apparatus connected with the attacking BM points of trajectory calculation, there are other priorities. The most important one is to create conditions as optimal as possible for the BM position and movement parameters determination.

In case the XBR (SBR, AN/SPY-1) tracking/guidance radar position is chosen in the vicinity of the BM plane of flight, it gets following consequences: azimuth of radius vector (axis of sight) $\dot{r}_i$ of warhead (value b_i) will be varying very slowly during the process of radar BM tracking. It is possible too, that this angle – due to the radar resolving power - will not vary at all. Consequently formulas (1) and (2) that are used for determination (in this particular case more precisely *approximation*) of the BM's flight plane position and orientation, are reduced to the form:

$$d_{0i} \approx 0 \quad \text{and} \quad b_{0i} \approx b_i - \frac{p}{2}. \tag{20}$$

It is obvious from mathematical formulas (13) and (14) used for the BM start and target positions calculations, that if the radar position and the BM flight plane are in the immediate closeness (i.e. $d_{0i} \to 0$), the calculations are impossible (zero in the denominator of the fraction).

It follows from the above mentioned, that it is desirable to select XBR (SBR, AN/SPY-1) radar position sideways from BM flight plane, approx. at the distance from 300 to 500 km.

Another very significant factor for GMD anti-ballistic missile system configuration is the time factor. Considering a generally very high BM's speed, there is a need for sufficient time to deal with BM threat successfully.

It can be seen in the Fig. 6, the XBR radar position is selected in the distance of 3 000 km from presumptive enemy's ballistic missiles starting point. In the Fig. 6, upper part, two ballistic missile trajectories are demonstrated as a result of modelling. Distance of 3 000 km was calculated for both cases: the low and non-energetically optimal trajectory, and the higher – energetically optimal trajectory. BM with 5 500 km range was simulated for the third case.
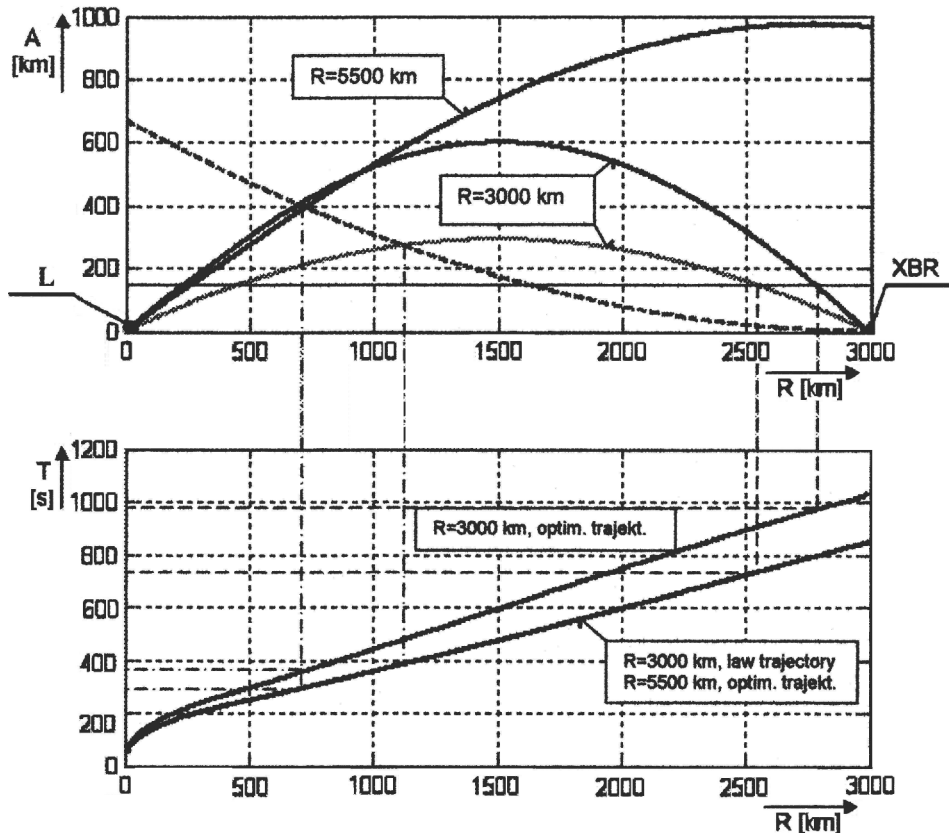

Fig. 6 GMD system functioning geometric and time relations

Dash line symbolizes the XBR radar range considering the earth curvature. In the Fig. 6, lower part, the times of BM flights are depicted depending on their actual distance from start position. The following facts can be read from graphs above:

- Middle-range (up to 3 000 km), or long-range (up to 5 500 km) ballistic missile, which is moving in energetic optimal trajectory (or in the similar route), will be discovered at earliest on altitude 400 km and in the distance circa 2 300 km from radar; for to act against such a BM (location and identification by XBR, flight plane determination, trajectory parameters calculation, anti-rocket board guidance system programming, launch and homing), there is at the disposal part of trajectory and corresponding time from its location to the point, where BM

(better its warhead/re-entry vehicle) falls to the altitude circa 150 km; this time is approximately 10 minutes for middle range BM (range 3 000 km). For BM range of 5 500 km the dispositional time is depending on anti-rocket launcher position too, and it makes 11 to 12 minutes at minimum;

- Middle-range (up to 3 000 km) ballistic missile which is moving in low trajectory, will be discovered at earliest on altitude 273 km and in the distance circa 1 900 km from radar; for to act against it only 5.5 minutes remain, which is not sufficient; it this case there will be probably a need to use data from satellites STSS system, or from the additional/forward radar.

It is obvious from the above mentioned that time factor is very important for GMD system functioning. The time required can be gained only if all components of surface and space system work perfectly. It means beginning from surface and space sensors (DSP, SBIRS STSS), BM command centre C4I, early warning ground radars UEWR, to fire systems. However, there is no 100% probability of successful GMD system functioning.

5. Conclusion

Principles and probable scenario of anti-ballistic rocket preparation to engage attacking BM warhead are described in this article. Middle and long-range ballistic missiles are analyzed and engagement in the exospheric phase of their flight is considered.

Mathematical formulas applied here allow expressing some crucial conclusions concerning GMD system appropriate configuration. There is a possibility to analyze the time relations influencing GMD system functioning.

References:

[1] KROULÍK, J and RŮŽIČKA, B. *Military Missiles* (in Czech). Praha : Naše vojsko, 1985.

[2] KHOL, R. *Anti-Ballistic Missile Defence and Czech Republic* (in Czech) [on line], Praha : Institute of International Relations Prague 2006 [cited 2008-03-05]. Available from
<http://se2.isn.ch/serviceengine/Files/EINIRAS/29251/ipublicationdocument_si ngledocument/FEDFECB7-69E2-43E6-8655-DA95528BD54B/cs/2006-10-Rakety.pdf>.

[3] VISINGR, L. *ALTBMD*: *European Pillar of Ballistic Missile Defence* (in Czech) [on line]. August 2006, [cited 2008-05-10]. Available from:
< http://www.natoaktual.cz/altbmd-evropsky-pilir-protiraketove-obrany-fm3-/na_analyzy.asp?c=A080505_092929_na_analyzy_m02 >.

[4] *STRATFOR Global Intelligence* [on line]. USA – Texas. [cited 2009-04-29]. Available from: <http://www.stratfor.com/images/northamerica/Missile-800.jpg> and http://www.milsatmagazine.com.

[5] KAUCKÝ, S. *Ballistic Missile Defence: Space-Based Infrared System SBIRS* (in Czech) [on line]. ATM - armády, technika, militaria, 2006, No 11, [cited 2008-01-20] Available from: <http://www.atmonline.cz/analyzy/pro/sbirs.htm>.

[6]   KAUCKÝ, S. *Ballistic Missile Defence: Radars XBR and SBX* (in Czech) [on
      line], Available from: <http://www.atmonline.cz/analyzy/pro/radars.htm>.

[7]   BRONSHTEIN, I. N. and SEMENDYAYEV, K. A. *Handbook of Mathematics*
      (in Russian). 10th ed., Moscow : Nauka, 1964.

[8]   DMITRIYEVSKIJ, A. A. et al. *Motion of Missiles* (in Russian) Moscow :
      Voyennoye izdatelstvo, 1968.

[9]   HAMTIL, I. *Ballistic Missile Defence* (in Czech). Brno : University of Defence,
      2008. ISBN 978-80-7231-584-0.

List of symbols

e	eccentricity of ellipse
g	acceleration of gravity [m·s^{-2}]
i	index
r	radius vector length [m]
t	time [s]
A	apogee
C	target position
G	gravitational constant $G = 6.67 \times 10^{-11}$ kg^{-1}·m^3·s^{-2}
M$_i$	BM's position at i^{th} measurement
M$_i''$	BM radius vector point of intersection with surface of the Earth
M_z	mass of the Earth; $M_z = 5.95 \times 10^{24}$ kg
O$_z$	Globe centre
R_z	Globe radius [m]
S	start position of BM
S_v	areal velocity of BM [m^2·s^{-1}]
b_i	azimuth of radius vector at the moment of ith measurement [rad], [°]
b_{0i}	orientation angle of BM flight plane [rad], [°]
c_{0i}	angle between directrix of point "P" and directrix of apogee [rad]
c_{Ci}	angle between directrix of point "P" and directrix of point C (target) [rad]
c_i	angle between directrix of point "P" and directrix of rocket [rad]
$\overset{\&}{}_i$	angular velocity [s^{-1}]
d_{0i}	radar angle deviation from BM flight plane [rad]
e_i	elevation of BM radius vector at i^{th} observation [rad], [°]
j_{ZC}	target geographical latitude [°]
l_{ZC}	target geographical longitude [°]
r_i	BM slant range (radius vector) [m]