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Response of Structure to Ballistic Load

P. Droppa[1*], M. Vančo[2] and V. Ferencey[3]

[1] Department of Mechanical Engineering, Armed Forces Academy, Liptovský Mikuláš, Slovakia
[2] Development – Services – Security – Innovations Ltd, Bratislava, Slovakia
[3] Department of Mechanics, Slovak Technical University Bratislava, Slovakia

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Abstract:

The paper describes briefly the problem of elastic-plastic shock wave propagation in solids. Such waves originate by acting of explosion waves or by high velocity impact of solid parts on the structure. Computational simulations are applied to the problem of propagation of shock waves in composite material reinforced by fibres. The fibres are supposed to be much stiffer than the matrix. The shock wave in such material is influenced by reflexion and refraction on the interface between fibres and the matrix and by the interaction of the waves. The shock wave is strongly damped in such composites in this way.

Keywords:

Ballistic load, shock wave propagation, fibre reinforced composite materials, shock wave damping, computational simulation.

1. Introduction

Explosion and high velocity impact are typical ballistic loads of structures (Fig. 1). In comparison to collision velocities of automobile crash, which are typically between 10 and 50 m/s, the impact problems deal with velocities of several hundred m/s. In the case of scenarios involving strain rates of $10^3\ s^{-1}$ and more, the evaluation and propagation of discontinuous compressive waves called shock waves, becomes more and more important with respect to the type and speed of deformation. The subsequent interaction of shock waves with boundaries of the solid and with each other produces a variety of stresses including those of shear, tension and compression. These stresses may result in severe damage to the structure integrity of the solid itself, as well as other structures in the immediate vicinity thereof. For example, a compressive shock

[*] *Corresponding author: Department of Mechanical Engineering, Armed Forces Academy of gen. M. R. Štefánik, Demänová 393, 031 01 Liptovský Mikuláš, Slovakia. E-mail: peter.droppa@aos.sk*

wave propagating in a solid will be reflected from a free surface. The reflected shock, which is a tension wave, may produce such severe stresses as to cause a complete separation of a portion of the solid in the region adjacent to the free surface. This effect, which is generally termed spall, may be not only destructive to the solid itself but also to other objects in the vicinity of the free surface due to high velocity separation of solid material adjacent to the free surface. At present, effort is made to find methods and means for attenuating shock waves propagating within a solid. When a portion of a shock wave propagating within a solid encounters a region of the solid having higher or lower shock wave speed than the region in which it was propagating, the encountering portion of the wave is scattered, typically being both reflected and refracted. An overall result of the total encounter may be a lateral spreading of kinetic energy.

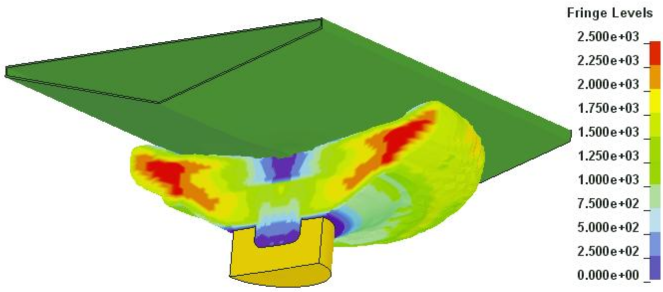

Fig. 1 Ballistic load of structure (frame)

Lightweight composite materials are great choice for future military, road and space vehicle systems because of their high specific modulus and high specific strength. However, there is a current lack of understanding of shock resistance of such materials and especially of composite materials, fibre reinforced materials (FRM) and foam materials under various shock loading conditions. The FRM are widely considered as new revolutionary materials. Due to complex topology of FRM, when shock waves propagate through it, the waves reflect, interact and refract at the material interfaces. As a result, a shock wave attenuation and late time dispersion became an interesting topic of research. With detailed understanding of the behaviour under various design conditions such as ballistic, shock and fire resistance, damage tolerance, manufacture conditions and cost effectiveness, one can improve the dynamic response of FRM by changing the structure of the composite.

Dynamic response of material is important in variety of military, as well as engineering applications [1-2]. Careful structural design taking into account the dynamic response has the potential to prevent catastrophic failure and save lives. Materials under high strain rates have much different mechanical properties than those measured under static or quasi-static loading conditions [3]. A shock is a discontinuity in stress, internal energy and material density. Shock-reshock and shock-release are important load conditions for spall strength of material. Spallation is the process of internal failure or rupture of continuous material through cavitation/de-cohesion due to stresses in excess of tensile strength of the material.

Shock wave propagation in heterogeneous materials is a complex matter. Phenomenon of material and geometric dispersion is yet poorly understood, as complex pattern is generated by a continuous interaction of compression and rarefaction waves generated by inter-face in material in homogeneities. The smaller the particle size, the greater the number of interfaces that interact with propagating stress waves and the higher the attenuation and dispersion. Interaction of leading shock front with secondary compression waves results in dissipation of the shock wave. The behaviour of material by shock wave loading can be simulated and studied by commercial software like LS DYNA, AUTODYN, DYTRAN, ABAQUS, PAM-SHOCK, etc., but the models require a large number of equations which have to be solved and resolved during propagation of the shock waves and so, highly efficient computers/supercomputers are necessary for modelling these complicated problems.

The aim of this paper is to contribute to a better understanding and modelling of scattering and dispersion of shock waves using commercial software. It is supposed that readers are familiar with basics of continuum mechanics and basic methods of simulation of classical problems of statics and dynamics using FEM and some mesh methods. Some special methods and corresponding governing equations used especially in simulations of shock propagation in solids are introduced.

Computational simulations are applied to the problem of propagation of shock waves in composite material reinforced by fibres. The fibres are supposed to be much stiffer than the matrix. The shock wave in such a material is influenced by reflexion and refraction on the interface between fibres and the matrix, as well as by the interaction of the waves. The shock wave is strongly damped in this way.

Another important part concerning the response of structure to ballistic load is not included into this paper because of space and time restrictions. Both readers and experts in modelling and numerical simulation of the problem can find details in a highly recommended literature [2].

2. Governing equations for shock waves propagation

We will not present here all governing equations for shock wave propagation, as they would contain basic relations of continuum mechanics, which the reader can find in one of the textbooks of continuum mechanics [3-5]. These equations are used to describe:

- the kinematics of solid continuum, the equations which present relation between displacements and corresponding displacement gradient for finite displacements in material and spatial description, strain tensors for finite strain formulation, strain measures and strain rate tensors,
- material and spatial time derivatives of deformations, velocity and velocity gradients,
- corresponding stress measures,
- formulation of equilibrium,
- conservation equations (conservation of mass, momentum and energy),

Thermodynamic laws give:

- the first law – the conservation of total energy,
- the second law – change in entropy,
- thermodynamic potentials – internal energy, enthalpy, Helmholtz and Gibbs free energy.

Further on, the constitutive equations which have to be thermodynamically consistent give the relation between stress and strain measures. Dynamic deformation processes, especially when shock wave formation is involved, are usually modelled by decomposed stress tensor. The decomposition splits the stress tensor into a deviatory tensor S_{ij} and a spherical hydrostat $p\delta_{ij}$, as:

$$\sigma_{ij} = S_{ij} - p\delta_{ij} \tag{1}$$

The usefulness of the decomposition results from the needed nonlinear character of equations of state (EOS) to describe shock waves. In general, a pure material can be solid, fluid or gas.

We will deal further with solids and only with special problems concerning shock waves propagation. More general problems are discussed in textbooks, e.g. [2-5]. Dynamic compressive behaviour of materials at strain rates in the regime of $10^6\ s^{-1}$ is typical for shock loading resulting elastic-plastic characteristic.

For most engineering applications involving equations of states, empirical relations with experimentally derived data are used. Their most simple representation is the so called linear equation of state which assumes isothermal processes and a linear pressure-volume or pressure-density relation. Via the bulk modulus *K*, the linear equation of state is formulated as:

$$p = K\varepsilon_{kk} = K(\rho/\rho_0 - 1) = K\mu \tag{2}$$

with the compression term μ describing the ratio of change in volume and density ρ from its initial state, ρ_0.

For isotropic materials, the bulk modulus *K* is linked to Young's modulus *E* and the shear modulus *G* via the Poisson ratio *v* by:

$$K = E/(3-6\nu) = 2G(1+\nu)/(3-6\nu) \tag{3}$$

meaning that the knowledge of any two other elastic constants provides the input needed for the linear equation (material dependent) in order to obtain the actual state.

Whenever the linear elastic region described in equation (2) is left, which is for example the case when a wide spectrum of pressure and energy shall be covered by the EOS, nonlinear relations are needed. A polynomial description of an equation of state can for instance be written as:

$$p = K_1\mu_1 + K_2\mu_2 + K_3\mu_3 + (B_0 + B_1\mu)\rho_0 e \tag{4}$$

where K_i and B_i are material constants usually defined separately for compression and expansion, respectively. An important difference from the linear equation (2) is marked by the energy dependence *e*, of the last term in (4). Whereas the linear equation is only a compression curve along an isotherm, the latter one can really be called equation of state in the sense of (4).

The observation of shock wave propagation can provide information to identify the material parameters in (4). The underlying theory is composed of the thermo-mechanics of shock waves, i.e. essentially the Rankine-Hugoniot equations providing a line of reference configurations on the state surface, used to identify the parameters K_i and an assumption on the pressure change of the Hugoniot-line along isochors, defining the constants B_i.

In the case of quasi-static loads, wave effects are not investigated since the duration of loading is long compared to the duration of multiple reflections throughout the structure. In addition, the resulting structural deformation and material state is not influenced in a comparable way by single wave transition. However, if the induced waves take the shape and amplitude of shock waves or the load speed is in the order of magnitude of the local sound speed, then wave effects and their propagation through the structure needs to be resolved in time and space.

Waves in solids are basically perturbations in the velocity field propagating through the continuum in different forms and at related different velocities. The propagating perturbation leads to wave form specific motion of the particles.

The most important wave forms in solids are:

- Longitudinal waves of compressive or tensile type which cause particle deflections along propagating direction. They are fastest wave forms in solids and are also called primary waves and the velocity is given by:

$$c_L = \sqrt{\frac{E}{\rho}} \tag{5}$$

- The next fastest waves are the *shear* or *secondary waves* causing particle motion perpendicular to the wave propagation. The speed is given by:

$$c_S = \sqrt{\frac{G}{\rho}} \tag{6}$$

  where G is shear modulus of material.
- Along the surface of solids propagate so called Rayleigh waves setting surface particles into elliptic motion and decaying in direction perpendicular to the surface.
- In structures of finite bending stiffness, flexural wave propagate upon dynamic loading.

In structures of complex form, a rather complicated combination of all basic wave forms can be observed.

Characteristic properties of all shock waves are extremely short rise times, as well as high pressure, density and temperature amplitudes. Basically, shock waves can arise as a sequence of both wave superposition and dispersion effects:

- If the source of a pressure disturbance is moving at a speed of sound of the surrounding medium or faster, superposition of the propagated disturbance and thus pressure waves leads to increased amplitudes and pressure gradients.
- In case of nonlinear pressure-density relations the corresponding dispersion effects lead to the formation of shock waves if faster wave components overtake earlier induced waves of lower propagation speed.

External dynamic compressive loads, initiated e.g. by impact or detonation, can possibly cause very strong waves with extremely short rise times inside structure. Superposition of different wave components is responsible for the steepening of the wave front. Superposition takes place as a consequence of dispersion, an effect that arises with nonlinear compressive behaviour.

In the initial elastic regime (p_0, V_0) compressive waves are propagated at the elastic wave speed [2], (see Fig. 2):

$$c_{elastic} = c_0 = -V\sqrt{\left.\frac{\partial p}{\partial V}\right|_0} \tag{7}$$

where V means specific volume (m^3/kg). As the load rises to higher pressures beyond the plastic threshold, the gradient and thus the propagation speed decrease drastically. Enhancement of pressure beyond p_1–V_1 state leads to a gradual increase of the modulus. From that turn around point onwards, pressure waves are initiated that propagate faster than others before. Consequently, a superposition of slower wave packages by faster ones with higher amplitude occurs.

In the light of these observations and with a mathematical description of the slopes in the *p–V* diagram of Fig. 2, conditions for the formation of shock waves can be formulated as:

$$\frac{\partial p}{\partial V} < 0 \tag{8}$$

$$\frac{\partial^2 p}{\partial V^2} > 0 \tag{9}$$

In materials with an elastic-plastic compressive behaviour according to Fig. 2, only loading conditions achieving pressures of p_1 or more can lead to shock waves. In gases and fluids, however, compressions shocks can arise from ambient pressures onwards since no regions with $\partial^2 p / \partial V^2 \leq 0$ exist.

Another necessary precondition for the shock formation is the rapid loading. Imagine a quasi-static load application to a pressure level indicated by p_1 in Fig. 2. Still, information about the applied load would be transported by waves at the sound speed defined by dispersion effects, i.e. depending on $\partial p / \partial V$. But time delay for each pressure increment along a certain equilibrium path would avoid the formation of a shock wave.

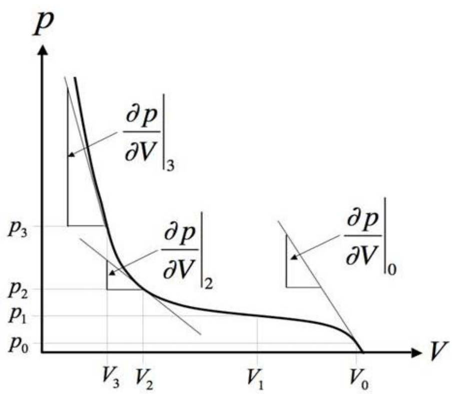

Fig. 2 Nonlinear compression curve of a solid elastic-plastic material allowing for dispersion driven shock waves (Reproduced with permission of Author [2])

Only if the load application is fast enough, the wave fronts of the faster packages keep up with the earlier wave fronts. The result is a steepened wave front and shorter rise times to higher pressures. Often, the wave components from the elastic regime are fast enough to enable the formation of a so-called *elastic precursor*. It is, however, also possible that even the elastic precursor is overtaken by very fast plastic waves. Whether this happens or not is only a matter of the load application speed and the achieved maximum pressure level.

A shock is a discontinuity of stress, temperature (or internal energy) and density. The shock propagation in a solid is derived under following assumptions: a shock is a discontinuous surface and has no apparent thickness; the shear modulus of material is assumed to be zero, such that it responds to the wave of a fluid (i.e. it is restricted to higher pressures); body forces and heat conduction at the shock front are negligible; there is no elastic-plastic behaviour, material does not undergo phase transformation. Then such material is described by the Rankine-Hugoniot conservation equations [2, 6, 7] which consist of the equation for mass:

$$\rho_0 v_s = \rho(v_s - v_p) \tag{10}$$

momentum:

$$(p - p_0) = \rho_0 v_s v_p \tag{11}$$

and energy:

$$p v_p = \frac{1}{2}\rho_0 v_s v_p^2 + \rho_0 v_s (e - e_0) \tag{12}$$

where ρ_0, ρ, v_s, v_p, p_0, p, e_0, and e are the initial density, density after impact, shock velocity, initial particle velocity, initial pressure, pressure after impact, initial internal energy and internal energy after impact, respectively. It is further assumed that the initial particle velocity and initial pressure are equal to zero in the equations above.

There are three equations, but five unknown variables: ρ, v_s, v_p, p, and e. Hence, an additional equation is needed to determine all parameters as a function of one of them. This fourth equation which can be conveniently expressed as the relationship between shock and particle velocities has to be experimentally determined by polynomial equation [8, 9]. For most materials the equation of state can be approximated as a linear relationship between the shock velocity and the particle velocity:

$$u_s = c_0 + S v_p \tag{13}$$

where S is the experimentally determined parameter and c_0 is the sound velocity in the material at zero pressure. This is the case even up to shock velocities around twice the initial sound speed c_0 and shock pressures in order of 100 GPa. If material undergoes large elastic-plastic deformations, the linear equation of state is no longer applicable and has to be modified (e.g. by polynomial of higher order). At high shock strengths some nonlinearity is apparent, particularly for non-metallic materials.

In elastic-plastic material the elastic wave speed is greater than the plastic sound speed so a shock wave separates out into two separate waves with the elastic wave called elastic precursor running ahead of the main plastic loading wave. If, however, the shock wave is so strong that its hydrodynamic shock speed is greater than the elastic sound velocity, the elastic and plastic loading waves coincide and no separation into a two wave structure occurs [2, 6].

Inelastic deformation behaviour of many materials depends on the deformation rate. For application to impact, strain rate dependent formulations of yield criteria, as well as other mechanical properties, are of interest. Various formulations are implemented in commercial codes. A frequently used model in high velocity impact is the Johnson-Cook strain, strain rate and temperature dependent yield model:

$$\sigma_Y = (\sigma_0 + B\bar{\varepsilon}_p^n)\cdot(1 + C\ln\dot{\bar{\varepsilon}}_p^*)\cdot(1 - T^{*m}) \tag{14}$$

with the equivalent plastic strain $\bar{\varepsilon}_p$ the equivalent plastic strain rate:

$$\dot{\bar{\varepsilon}}_p^* = \frac{\dot{\bar{\varepsilon}}_p}{\dot{\varepsilon}_0} \tag{15}$$

normalized to $\dot{\varepsilon}_0 = 1\,\mathrm{s}^{-1}$ and with the homologous temperate:

$$T^* = \frac{T - T_A}{T_M - T_A} \tag{16}$$

where T_A is the ambient or reference temperature and T_M the melting temperature.

The five parameters σ_0, B, n, C, m allow for a description of strain hardening, as well as strain rate and temperature dependency. The first three of them can be derived from quasi-static tension tests at constant strain rate. The strain rate parameter C is identified through dynamic tension tests at varying strain rates.

Experimental methods to characterize material behaviour and to identify parameters needed in the mathematical formulations of constitutive laws are sometimes limited. Numerical simulation can be applied in such situations, provided a sufficient basic knowledge of the investigated material is at hand. If for example the inelastic behaviour of a composite material is of interest and the individual material properties of the components, e.g. matrix and fibre or aggregate (e.g. in aluminium foam [10]), are sufficiently characterized, then a meson-mechanical simulation of the composite material can be performed to numerically identify composite properties.

There is a big variety of polymeric materials which are macromolecular configuration of long, randomly twisted, possibly entangled and interconnected chains. A material specific temperature, the so called glass transition temperature, separates glassy state, i.e. a solid phase, from the rubbery which can be described as a viscous fluid type phase. In order to distinguish polymer behaviour from that is observed with metals, different sections of typical stress-strain curves need new types of mathematical formulations. A specific understanding of the processes at scale and its influence on the macro-phenomena is necessary [2]. Pronounced localization of strain and distinct differences under tensile, compressive or shear loading is evident. The strain localization leads to very different results in stress-strain curves depending on the size of the measured zone.

Deformation process can lead to fundamental changes in properties of materials (failure, crack opening, multiple fragmentation and phase changes). However, these problems are out of the scope of this paper.

Impact processes are dominated by transient stress and strain states. Typical duration for the deformation processes are milliseconds. Spatial and time discretization must be fine enough to correctly simulate the process.

Hydro-codes, also called wave-propagation-codes, are typical class of numerical tool for simulation of impact. Numerical methods, the Finite Element Method (FEM), Finite Volume Method (FVM) and Mesh Free Methods, especially Smoothed Particle Hydrodynamics (SPH)) and their combination (coupling) are mostly used for the simulation.

Deformable structures subject to blast effects are in many ways a real challenge to numerical simulation. Both the fluid dynamical and structural dynamic regimes need to be modelled carefully to cover all relevant effects.

Computational simulations are often very demanding and we will show only the most complicated cases, which are of great interest for the design of increasing safety of vehicles against explosion and penetration of missiles and splitters through the structure.

3. Computational simulations of shock waves in composite materials

As it has been shown in the previous section, the shock wave velocity is influenced by material properties, modulus of elasticity, material density, temperature, etc. Modern composite materials are reinforced by particles, fibres or layers from materials of stiffness considerably higher than that of the matrix. Such material is important for many applications, as its stiffness and strength are often much higher than that of the homogeneous material [11-13]. Also the dynamical properties of such materials differ from homogeneous materials by much higher damping which is important for impact by low and high velocities, but also by other loading conditions. Computational simulations described below document the great importance of the damping in materials reinforced by fibres.

In the following example the composite material with modulus of elasticity and density equal to 210 GPa and 7 830 kg/m^3, respectively, is reinforced by straight fibres regularly distributed parallel to the upper surface (see Fig. 3).

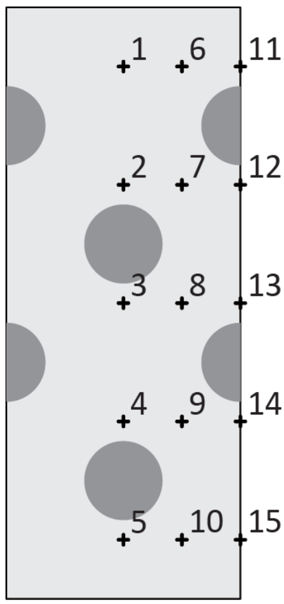

Fig. 3 Matrix reinforced by fibres

The modulus of elasticity of fibres is larger than that of the matrix. Radius of fibres is 1 mm and their centres are in plains 3 mm far from each other so that their volume is 35 % of the composite. The loading of the material is perpendicular to the surface and it is increasing from zero to 0.0315 GPa in 0.05 μs and decreasing back to zero in the same time. It is a 2D problem (plain strain) and computational simulations were performed in LS DYNA [14].

Figures 4 to 15 show the effective (von Misses) stresses in time after the shock achieving the surface of the material all in points 1 to 15 as defined in Fig. 3. The dark colour corresponds to the point close to the surface and the light to the other point below the first one. From the figures one can find the movement of the front, as well as the maximum of the stress in time.

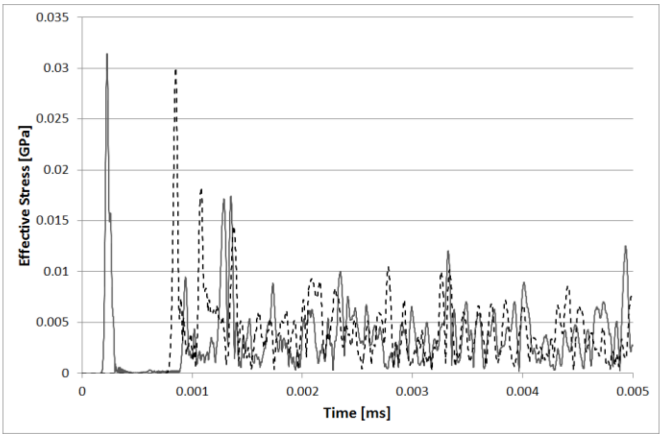

Fig. 4 Effective stress in points 1 and 2

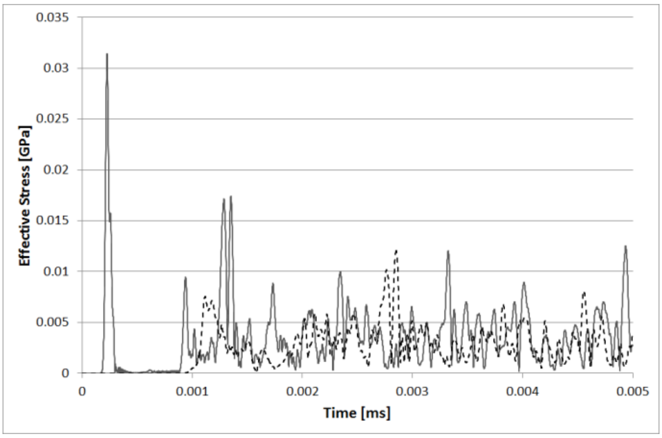

Fig. 5 Effective stress in points 1 and 3

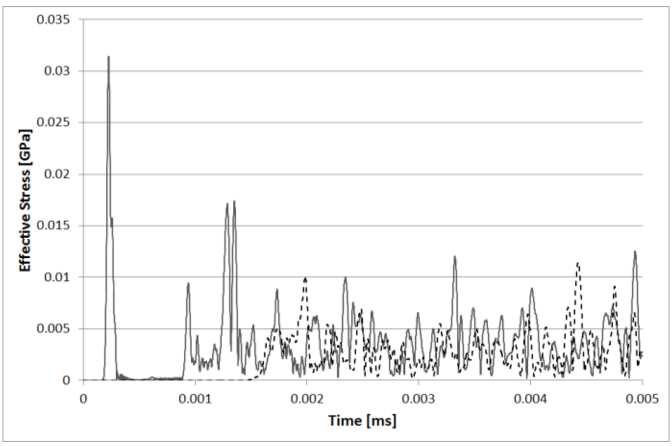

Fig. 6 Effective stress in points 1 and 4

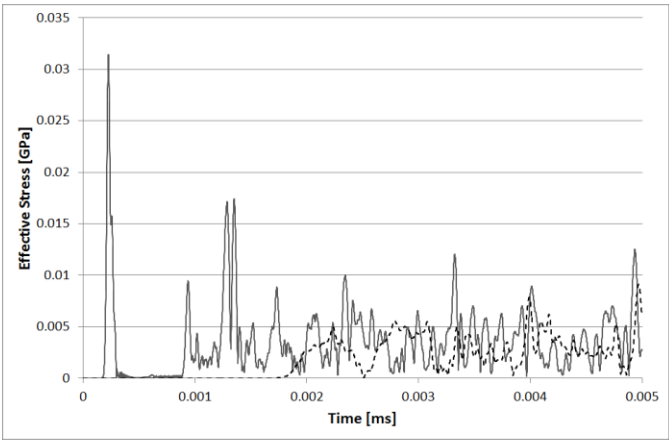

Fig. 7 Effective stress in points 1 and 5

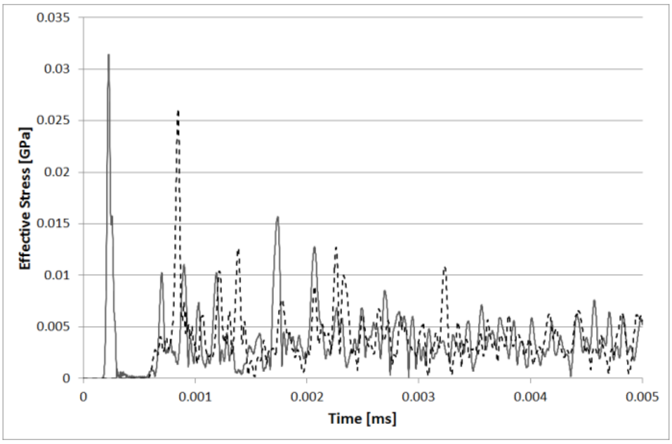

Fig. 8 Effective stress in points 6 and 7

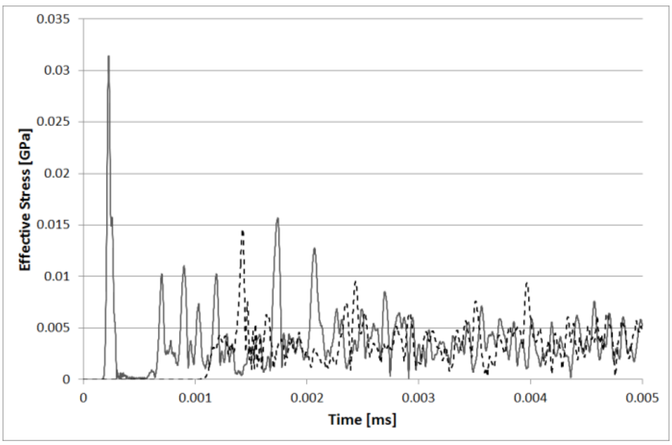

Fig. 9 Effective stress in points 6 and 8

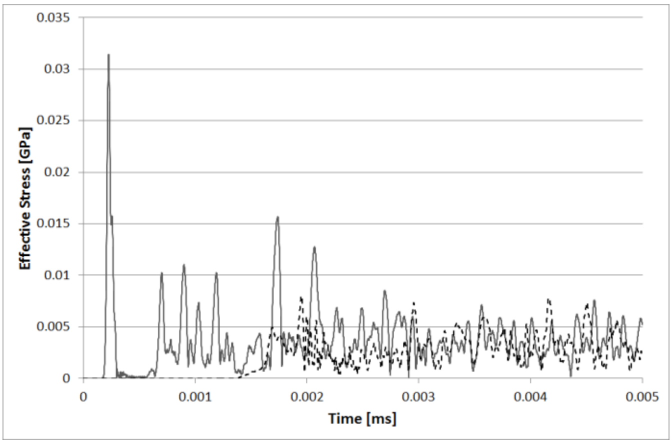

Fig. 10 Effective stress in points 6 and 9

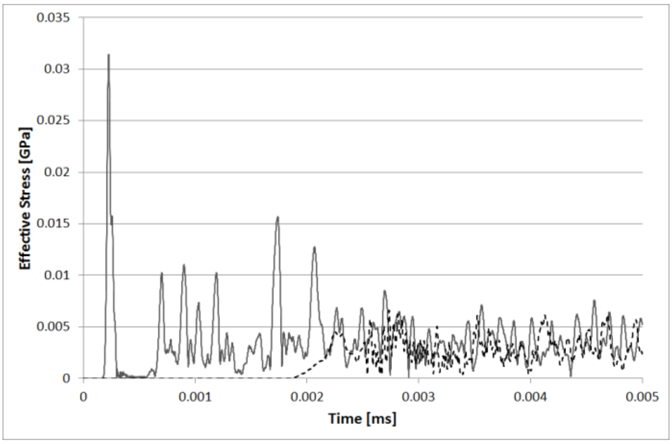

Fig. 11 Effective stress in points 6 and 10

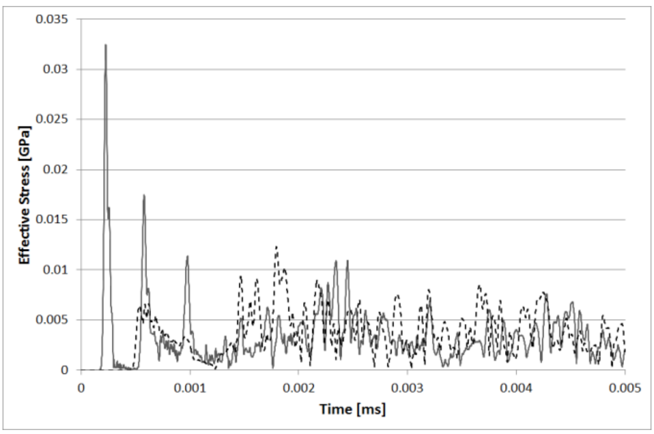

Fig. 12 Effective stress in points 11 and 12

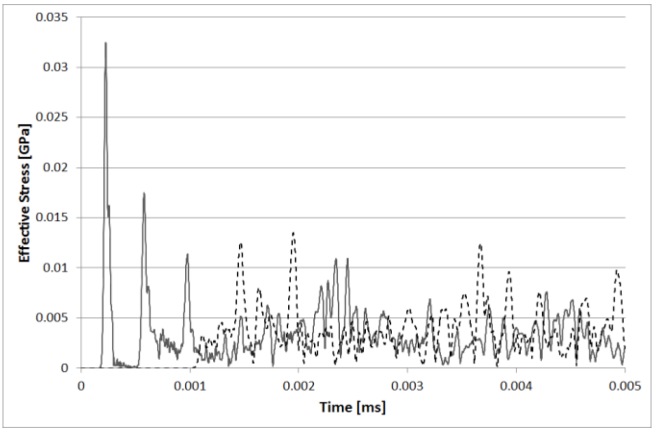

Fig. 13 Effective stress in points 11 and 13

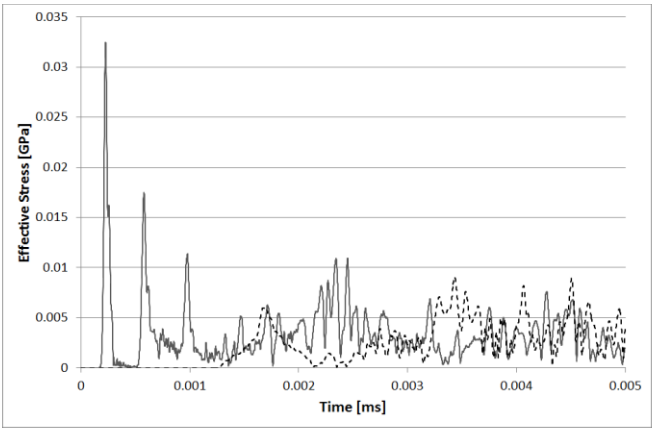

Fig. 14 Effective stress in points 11 and 14

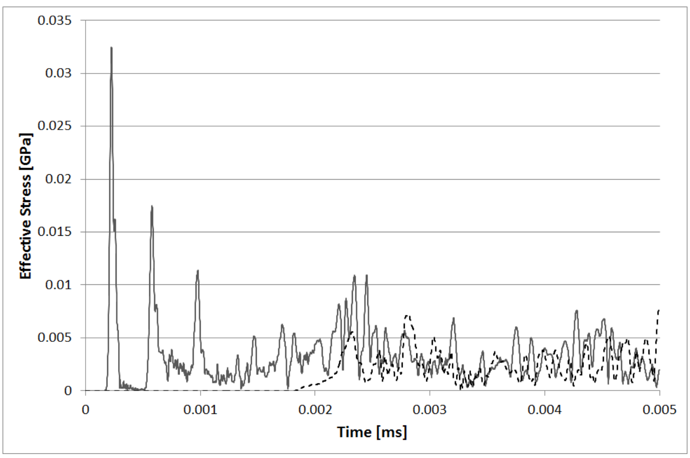

Fig. 15 Effective stress in points 11 and 15

Table 1 contains effective stresses in the front of the wave and maximal stresses in all points and corresponding time when they reach the corresponding point.

Tab. 1 Effective stresses in points

Point	Front wave stress [GPa]	Time schedule of front wave stress [ms]	Maximum stress [GPa]	Time schedule of maximum stress [ms]
1	0.03140199	0.000225904	0.03140199	0.000225904
2	0.02988118	0.000849899	0.02988118	0.000849899
3	0.00753032	0.001116990	0.01208089	0.002851820
4	0.00407013	0.001650970	0.01143547	0.004426780
5	0.00206880	0.003210553	0.00910127	0.004969670
6	0.03140199	0.000225904	0.03140199	0.000225904
7	0.00275854	0.000641830	0.02615910	0.000846927
8	0.00310550	0.001163910	0.01466076	0.001424850
9	0.00493289	0.001689820	0.00800138	0.001951820
10	0.00447741	0.002253940	0.00662935	0.002734840
11	0.03244800	0.000225904	0.03244800	0.000225904
12	0.00638929	0.000535884	0.01228444	0.001794920
13	0.00328968	0.001114870	0.01349361	0.001950970
14	0.00596257	0.001673900	0.00905640	0.003433990
15	0.00557343	0.002325920	0.00717925	0.002814880

At the beginning the wave propagates parallel to the surface without any interaction (Fig. 16). After reaching first fibres the wave reflects from the fibre and interact with the reflected part, however, the front of the wave is still expressive (Fig. 17). After the front of wave continues to propagate to lower part under the surface, the maximum in corresponding point is not as high as in many other points closer to the surface. See Fig. 18, which corresponds to the moment when the effective stress is maximal in point 5, but the stresses in points closer to the upper surface are larger because of complicated interactions of the waves.

4. Summary

The results are summarised in Tables 2 and 3. Table 2 gives an average of maximal stress in the planes parallel to surface and going through indicated points and Table 3 gives the damping of these stresses between corresponding planes. The last row in the table gives the damping of the maximal stress owing to the interactions with the fibres related to the intensity of the input load.

Tab. 2 Average maximal stress in planes below the surface

Cut points	Stress [GPa]
1 – 11	0.04254
2 – 12	0.03031
3 – 13	0.01633
4 – 14	0.01173
5 – 15	0.00904

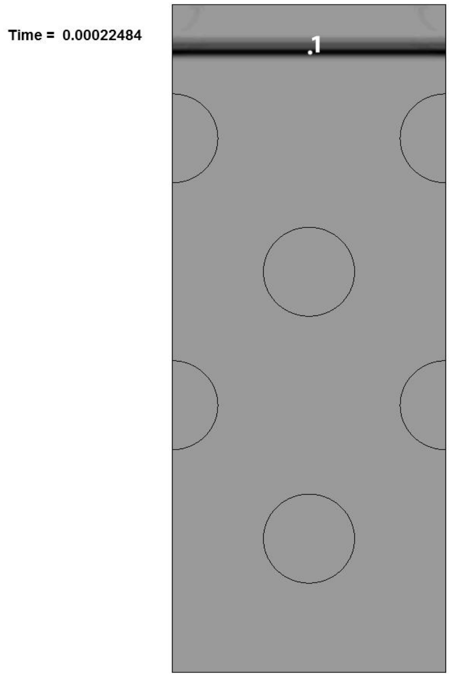

Fig. 16 Moment of the maximal effective stress passing point 1

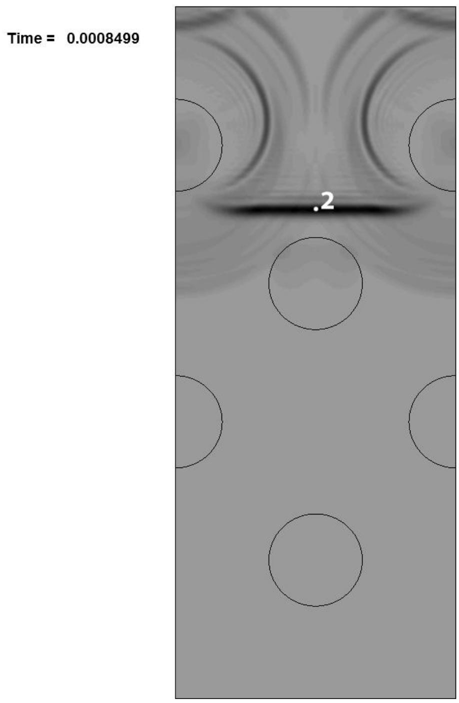

Fig. 17 Moment of the maximal effective stress passing the point 2

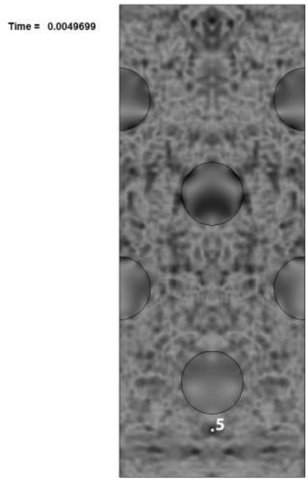

Fig. 18 Moment of the maximal effective stress passing point 5

Tab. 3 Damping of the maximal stress

Damping	Absorption [-]
1 – 3	2.607
2 – 4	2.583
3 – 5	1.807
1 – 2	1.405
2 – 3	1.856
3 – 4	1.392
4 – 5	1.298
1 – 5	4.712
Max. 1 – 5	4.503

5. Conclusion

Presented computational models do not contain any other damping except of the interaction of the shock wave with fibres (each material contains some imperfections in the structure and in material properties and so, there is some material damping also in homogeneous material) and show as the reinforcing fibres because of very different material properties of both matrix and fibres result in very efficient damping of shock waves and thus such composite can be very efficient in defence against explosion.

The shock wave in homogeneous material is not influenced by propagation through material and only when it is reflected on the boundaries there is an interaction with propagated wave [2, 12-17]. On the other side, there is a very complicated interaction of the wave by reflection and refraction on the interface between softer matrix and stiffer fibres leading to strong damping of the shock wave.

The computational simulations allow very effectively and without extensive and expensive experiments to study the behaviour of composite materials from all points of view, the material structure topology, material properties of components, percentage of reinforcement, etc.

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