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Mathematical Model of a Gas-Operated Machine Gun

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Abstract:

The article describes a thermodynamic mathematical model of internal ballistics in the barrel and in the gas cylinder of a gas-operated gun. In addition, this thermodynamic mathematical model deals with the mass flow of gas through the ring around the piston into the atmosphere. The thermodynamic mathematical model and the solution algorithm are validated and verified experimentally on the example of a 7.62 mm UK-59 machine gun and 7.62×54 R ammunition. The conclusions of this paper are applicable to the calculation and design of the machine gun gas propulsion structure for similar weapons with dust gas extraction.

Keywords:

automatic weapons, gas-operated gun, internal ballistic

1 Introduction

In gas-operated automatic weapons, part of the powder gas is removed from the barrel bore by means of a gas port. The principle of the drive is shown in Fig. 1. It consists of a piston, a gas port and a gas cylinder. After the initiation of the shot, the projectile starts to move and when the bottom of the projectile is behind the gas port, the gases enter the cylinder. The piston is controlled by the pressure of the expanding gases in the cylinder. The pressure is usually transmitted to the breech of the gun, which is set in motion by this pressure and performs the functional cycle of an automatic weapon.

The advantage of the principle is that the structure is simple and the amount of gas entering the gas cylinder can be adjusted. The value of the gas pressure in the gas cylinder influences the correct function of the gun. There are several small caliber automatic weapons that use gases drawn from ports in the bore of the barrel to power the automatic system, such as the AK-47, M16A1, AR-15, RPK, RPD, PKMS, UK-59, SA-58, BREN, etc.

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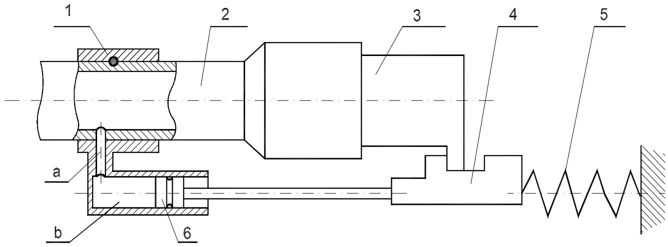

Fig. 1 Scheme of gas-operated machine gun
1 – gas block, 2 – barrel, 3 – bolt, 4 – bolt carrier, 5 – return spring, 6 – piston, a – gas port, b – gas cylinder

A gas-operated automatic weapon is a firearm in which a portion of the powder gas is used to control the movement of the breech. The value of the gas pressure must be sufficient to cause the breech movement. Thanks to its kinetic energy, breech performs all the important operations: ejecting the empty cartridge case, preparing the trigger mechanism and loading a new cartridge into the barrel. This pressurized gas in the gas cylinder impinges on the piston forehead to provide motion for breech and breech carrier, for more details, refer to [1]. There are several analytical methods to determine the gas pressure in the gas cylinder such as e.g. the methods presented in [1] and [4]. However, the amount of propellant gas charged into the gas cylinder is small. These methods consider that propellant gases do not affect the law of pressure, temperature, mass flow, and velocity projectile in the bore, etc. So, when solving the interior ballistics, it is not needed to consider the propellant gases taken from ports in the barrel, and the results of the interior ballistics are used to determine pressurized gas and motion of piston. In [5] the gas flow between the barrel and the gas cylinder is considered to be a one-dimensional flow. In this case, only the gas flow from the barrel into the gas cylinder is considered.

In addition, the pressure in the gas cylinder can be calculated based on the empirical and semi-empirical methods, which have been presented in [6]. These methods can be easily used and simply calculated. However, the accuracy of the results of calculations is not high.

In the last years, besides the analytical methods, the thermodynamic properties of the propellant gases inside the barrel and inside the gas cylinder were also studied by numerical methods. Jevtic et al. [7] studied the change of the thermodynamic properties in the gas cylinder and the movement of the piston of a 20 mm gun. Florio [8] performed the study of flow characteristics in the barrel and in the gas chamber of a M16A1 rifle.

This paper aims to develop a novel thermodynamic model that correctly and completely describes the internal ballistic cycle in the barrel and the phenomenon which occurs in the gas cylinder, starting with ignition, combustion of the propellant charge, the process of the projectile moving inside the barrel, the process of the propellant gases entering to the gas cylinder when the projectile has passed the gas port, the process of piston movement in the gas cylinder, until the gas pressure in the barrel rapidly drops to atmospheric pressure for all types of gas-operated automatic weapons. This model is based on laws, including the first law of thermodynamics; the equations of state; the law of conservation of mass; the burning rate law of propellant; equations of motion of a projectile; the relative quantity of burnt-out propellant. To measure gas pressure in the gas barrel and in the cylinder with different diameter of the gas port, an experiment on the machine gun UK-59 was set up and carried out. The experimental results were compared with analytical results.

2 Computational Model

2.1 Physical Model

The mathematical thermodynamic model is based on the laws of thermodynamics according to the gun scheme as shown in Fig. 2.

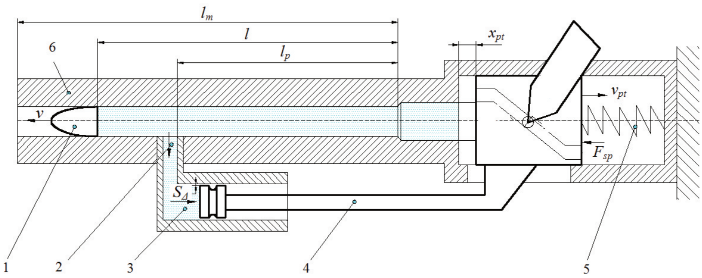

Fig. 2 Physical model of gas-operated machine gun
1 – projectile, 2 – gas port, 3 – gas cylinder, 4 – piston, 5 – return spring, 6 – barrel, l_{m} *– barrel length,* l_{p} *– position of gas port,* l *– trajectory projectile,* x_{pt} *– breech displacement,* v_{pt} *– breech velocity,* F_{sp} *– return spring force*

The whole process of firing and gas flowing into the gas cylinder is explained in detail in [1] and [6].

2.2 Mathematical Model

In the establishment of the mathematical thermodynamical model, we use the following assumptions:

- the propellants are burned according to the geometric rules of combustion and the combustion rate is as follows: $u = u_1 p$,
- the propellants burn at the same pressure, which is equal to the ballistic pressure p,
- the projectile moves due to the average pressure in the barrel,
- the movement of the projectile through the gas port is instantaneous; the gradual uncovering of the gas port is not considered,
- the return spring characteristic is linear,
- except for the return springs, which are elastic, all parts in the physical model are rigid,
- the model is an open thermodynamic system,
- the heat transfer between the walls surface and inside of the barrel and the cylinder is neglected,
- the specific heat capacity at constant volume c_v and the specific heat capacity at constant pressure c_p are average values and do not change over time.

The mathematical model describes the thermodynamic process of the internal ballistics of gas-operated automatic weapons, i.e., the combustion process, the gas generation process, the gas removal process from the barrel bore and the gas expansion process in the gas cylinder. The system of differential equations and algebraic equations is set up as follows:

a. System equations of the burning rate of propellant gases $t \in (0, t_k)$ [9]

$$\frac{dz}{dt} = \begin{cases} \dfrac{p}{I_k} & \text{when } 0 < e < e_1 \\ 0 & \text{when } e = e_1 \end{cases} \tag{1}$$

$$\frac{d\psi}{dt} = \begin{cases} \left(\kappa_c + 2\kappa_c \lambda z + 3\kappa_c \mu z^2\right)\dfrac{dz}{dt} & \text{when } 0 < t \le t_k \\ 0 & \text{when } t > t_k \end{cases} \tag{2}$$

where
p – the spatial averaged pressure in the barrel,
I_k – the total impulse of the propellant gases,
e_1 – the thickness burned of propellant,
e – the thickness burned of propellant at the time t,
ψ – the relative burnt mass of the propellant,
κ_c, μ, λ – the shape characteristic quantity of fast burning propellant,
z – the relative burnt thickness of the propellant,
t_k – the time when the propellant burned out.

b. System equations of the projectile movement $t \in (0, t_m)$ [9]

$$\frac{dv}{dt} = \begin{cases} 0 & \text{when } p \le p_0 \\ \dfrac{Sp}{\varphi m_p} & \text{when } p > p_0 \end{cases} \tag{3}$$

$$\frac{dl}{dt} = v \tag{4}$$

where
v – the velocity of projectile,
m_p – the mass of projectile,
S – the cross-sectional area of the bore,
p_0 – the projectile starting pressure,
φ – the fictitious factor,
l – the travel of projectile in the barrel,
t_m – the time when the projectile passes the muzzle.

c. The equation of determining the gas flow through the gas ports from the barrel into the gas cylinder and vice versa

- Period $l \le l_p$:

$$\frac{dm_{cg}}{dt} = 0 \tag{5}$$

where
l_p – the distance from gas ports to the initial position of projectile,

$\mathrm{d}m_{cg}/\mathrm{d}t$ – the mass flow rate of gas through gas port.

- Period $l > l_p$:

depending on the value of the gas pressure in the barrel (p) and the pressure of the gas in the gas cylinder (p_{cg}), the gas product can be flowed from the barrel into the cylinder or vice versa.

- Period $p > p_{cg}$:

in this case, the gas product flows from the barrel into the cylinder. The equation of determining the gas flowed through the gas ports from the barrel into the gas cylinder has the form [1]:

$$\frac{\mathrm{d}m_{cg}}{\mathrm{d}t} = \begin{cases} \varphi_1 S_0 K_0(\kappa)\dfrac{p}{\sqrt{rT}} & \text{when } \dfrac{p}{p_{cg}} \geq \left(\dfrac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}} \\ \varphi_1 S_0 \dfrac{p}{\sqrt{rT}}\sqrt{\dfrac{2\kappa}{\kappa-1}\left[\left(\dfrac{p_{cg}}{p}\right)^{\frac{2}{\kappa}} - \left(\dfrac{p_{cg}}{p}\right)^{\frac{\kappa+1}{\kappa}}\right]} & \text{when } 1 < \dfrac{p}{p_{cg}} < \left(\dfrac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}} \end{cases} \tag{6}$$

- Period $p < p_{cg}$:

the gas product flows from the gas cylinder into the barrel. The equation of determining the gas flowed through the gas ports from the gas cylinder into the barrel has the form [1]:

$$\frac{\mathrm{d}m_{cg}}{\mathrm{d}t} = \begin{cases} -\varphi_2 S_0 K_0(\kappa)\dfrac{p_{cg}}{\sqrt{rT_{cg}}} & \text{when } \dfrac{p_{cg}}{p} \geq \left(\dfrac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}} \\ -\varphi_2 S_0 \dfrac{p_{cg}}{\sqrt{rT_{cg}}}\sqrt{\dfrac{2\kappa}{\kappa-1}\left[\left(\dfrac{p}{p_{cg}}\right)^{\frac{2}{\kappa}} - \left(\dfrac{p}{p_{cg}}\right)^{\frac{\kappa+1}{\kappa}}\right]} & \text{when } 1 < \dfrac{p_{cg}}{p} < \left(\dfrac{\kappa+1}{2}\right)^{\frac{\kappa}{\kappa-1}} \end{cases} \tag{7}$$

where

φ_1 – the discharge coefficient of gases flowing through the gas vent from barrel bore to the gas cylinder,

φ_2 – the discharge coefficient of gases flowing through the gas vent from gas cylinder to the barrel bore,

$K_0(\kappa) = \left(\dfrac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\sqrt{\dfrac{2\kappa}{\kappa+1}}$ – the equation of exponent of adiabatic expansion,

κ – the Poisson constant (the ratio of the specific heats) of propellant gases,

T – the temperature of gas product in the barrel,

T_{cg} – the temperature of gas product in the gas cylinder,

r – the specific gas constant of propellant gases,

S_0 – the cross-sectional area of gas ports connected the barrel bore with gas cylinder.

- Period $p = p_{cg}$:

$$\frac{\mathrm{d}m_{cg}}{\mathrm{d}t} = 0 \tag{8}$$

d. The equation of determining the gas flow through annulus around the piston to the atmosphere

Due to the pressure difference in the gas cylinder and in atmosphere, the gas flows through annulus around the piston to the atmosphere. The flow gas propellant in this case is considered critical. This mass flow rate of the flow gas propellant is calculated by Eq. (9) [1]:

$$\frac{\mathrm{d}m_{\mathrm{atm1}}}{\mathrm{d}t} = \varphi_3 S_\Delta K_0(\kappa)\frac{p_{\mathrm{cg}}}{\sqrt{rT_{\mathrm{cg}}}} \tag{9}$$

where:
φ_3 – the discharge coefficient of gases flowing through the annulus,
$\mathrm{d}m_{\mathrm{atm1}}/\mathrm{d}t$ – the mass flow rate of the gas flowing through annulus around the piston to the atmosphere,
S_Δ – the area of the annulus between piston and gas cylinder.

e. The equation of determining the gas flow from the barrel to the atmosphere when the projectile passes the muzzle

After the projectile comes out of the barrel, because the pressure of the gas product inside the barrel is greater than the atmospheric pressure, the phenomenon of gushing from the bore of the barrel to the environment occurs. This process ends when the pressure in the barrel is equal to atmospheric pressure.

- Period $p > p_{\mathrm{atm}}$:

The mass flow rate of the gas through the muzzle to the atmosphere is calculated by Eq. (10) [1]:

$$\frac{\mathrm{d}m_{\mathrm{atm2}}}{\mathrm{d}t} = \begin{cases} \varphi_4 S K_0(\kappa)\dfrac{p}{\sqrt{rT}} & \text{when} \quad t > t_{\mathrm{m}} \\ 0 & \text{when} \quad t \le t_{\mathrm{m}} \end{cases} \tag{10}$$

where
$\mathrm{d}m_{\mathrm{atm2}}/\mathrm{d}t$ – the mass flow rate of the gas through the muzzle to the atmosphere,
p_{atm} – the atmospheric pressure,
φ_4 – the discharge coefficient of flowing gases through the muzzle.

- Period $p = p_{\mathrm{atm}}$

$$\frac{\mathrm{d}m_{\mathrm{atm2}}}{\mathrm{d}t} = 0 \tag{11}$$

f. The equation of state in the barrel:

$$pV = mrT \tag{12}$$

where
V – the instantaneous volume of the gas product in the barrel,
m – the mass of the gas product in the barrel (space in the barrel after the projectile).

$$\begin{cases} m=\omega\psi\ ;\ V=V_0+Sl-\dfrac{\omega}{\delta}(1-\psi)-\alpha\omega\psi & \text{when}\quad l\le l_\mathrm{p} \\ m=\omega\psi-m_\mathrm{cg}\,;\ V=V_0+Sl-\dfrac{\omega}{\delta}(1-\psi)-\alpha(\omega\psi-m_\mathrm{cg}) & \text{when}\quad l_\mathrm{p}<l\le l_\mathrm{m} \\ m=\omega\psi-m_\mathrm{cg}-m_\mathrm{atm2}\ ;\ V=V_0+Sl_\mathrm{m} & \text{when}\quad l>l_\mathrm{m} \end{cases} \tag{13}$$

where
V_0 – the initial volume of the barrel,
δ – the power density of propellant,
α – the covolume of powder gases,
ω – the mass of propellant,
l_m – the length of the bore.

g. Energy balance equation in the barrel

- Period $t \le t_\mathrm{m}$

the energy equation in the barrel is based on the first law of thermodynamics [10]:

$$\mathrm{d}Q=\mathrm{d}U+\sum\mathrm{d}L=\mathrm{d}U+\mathrm{d}E_\mathrm{pr}+\mathrm{d}H_\mathrm{cg} \tag{14}$$

where
Q – the energy of propellant,
U – the internal energy of gas in the barrel,
E_pr – the kinetic energy of the projectile,
H_cg – the enthalpy of the mass of gas product exchanged between the barrel and the gas cylinder.

$$\mathrm{d}Q=\left(\dot m_+\mathrm{d}t\right)c_\mathrm{v}T_\mathrm{v}=c_\mathrm{v}T_\mathrm{v}\dot m_+\mathrm{d}t \tag{15}$$

where
T_v – the propellant ignition temperature,
$\mathrm{d}m_+/\mathrm{d}t$ – the mass gas flow rate of the propellant burning.

$$\dot m_+=\frac{\mathrm{d}m_+}{\mathrm{d}t}=\omega\frac{\mathrm{d}\psi}{\mathrm{d}t} \tag{16}$$

$$\mathrm{d}U=\mathrm{d}\left[\left(m_+-m_\mathrm{cg}\right)c_\mathrm{v}T\right]=c_\mathrm{v}\left[\left(\dot m_+-\dot m_\mathrm{cg}\right)T\mathrm{d}t+\left(m_+-m_\mathrm{cg}\right)\dot T\mathrm{d}t\right] \tag{17}$$

$$\mathrm{d}E_\mathrm{p}=\mathrm{d}\left(\frac{\varphi m_\mathrm{p}v^2}{2}\right)=\varphi m_\mathrm{p}v\dot v\mathrm{d}t \tag{18}$$

$$\mathrm{d}H_\mathrm{cg}=\begin{cases}0 & \text{when}\ \ p=p_\mathrm{cg}\ \text{or}\ l\le l_\mathrm{p}\\ \dot m_\mathrm{cg}c_\mathrm{p}T\mathrm{d}t & \text{when}\ \ p>p_\mathrm{cg}\ \text{and}\ l>l_\mathrm{p}\\ \dot m_\mathrm{cg}c_\mathrm{p}T_\mathrm{cg}\mathrm{d}t & \text{when}\ \ p_\mathrm{cg}>p\ \text{and}\ l>l_\mathrm{p}\end{cases} \tag{19}$$

Introducing Eqs (15) and (17)-(19) into Eq. (14), it yields:

$$\dot T=\frac{\mathrm{d}T}{\mathrm{d}t}=\frac{1}{m_+-m_\mathrm{cg}}\left[T_\mathrm{v}\dot m_+-\left(\dot m_+-\dot m_\mathrm{cg}\right)T-\frac{1}{c_\mathrm{v}}\left(\varphi m_\mathrm{p}v\dot v+\dot H_\mathrm{cg}\right)\right] \tag{20}$$

- Period $t > t_m$:

the energy equation in the barrel is based on the first law of thermodynamics [10]:

$$\mathrm{d}Q = \mathrm{d}U + \sum \mathrm{d}L = \mathrm{d}U + \mathrm{d}H_{\mathrm{cg}} + \mathrm{d}H_{\mathrm{atm2}} \tag{21}$$

where

$\mathrm{d}H_{\mathrm{atm2}}/\mathrm{d}t$ – the enthalpy rate of the mass of gas product flow from the barrel to the atmosphere when the projectile passes the muzzle.

$$\mathrm{d}U = \mathrm{d}\left(mc_{\mathrm{v}}T\right) = c_{\mathrm{v}}\left[\left(\dot{m}_{+} - \dot{m}_{\mathrm{cg}} - \dot{m}_{\mathrm{atm2}}\right)T\mathrm{d}t + \left(m_{+} - m_{\mathrm{cg}} - m_{\mathrm{atm2}}\right)\dot{T}\mathrm{d}t\right] \tag{22}$$

$$\mathrm{d}H_{\mathrm{atm2}} = H_{\mathrm{atm2}}\mathrm{d}t = \dot{m}_{\mathrm{atm2}}c_{\mathrm{p}}T\mathrm{d}t \tag{23}$$

Introducing Eqs (15), (19), (22), and (23) into Eq. (21) we obtain:

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{1}{m_{+} - m_{\mathrm{cg}} - m_{\mathrm{atm2}}}\left[T_{\mathrm{v}}\dot{m}_{+} - \left(\dot{m}_{+} - \dot{m}_{\mathrm{cg}} - \dot{m}_{\mathrm{atm2}}\right)T - \frac{1}{c_{\mathrm{v}}}\left(\dot{H}_{\mathrm{cg}} + \dot{H}_{\mathrm{atm2}}\right)\right] \tag{24}$$

At the time the propellant has burned:

$$m_{+} = \omega \qquad \text{and} \qquad \dot{m}_{+} = 0 \tag{25}$$

h. <u>The equation of state in the gas cylinder</u>:

$$p_{\mathrm{cg}}V_{\mathrm{cg}} = m_{\mathrm{c}}rT_{\mathrm{cg}} \tag{26}$$

where

m_c – the mass of the gas product in the gas cylinder

$$m_{\mathrm{c}} = m_{\mathrm{cg}} - m_{\mathrm{atm1}} + m_{\mathrm{c0}} \tag{27}$$

m_{c0} – the initial mass of the gas in the gas cylinder,

V_{cg} – the instantaneous volume of the gas product in the cylinder in front of drive piston.

$$V_{\mathrm{cg}} = V_{\mathrm{cg0}} + S_{\mathrm{c}}x_{\mathrm{pt}} - \alpha\left(m_{\mathrm{cg}} - m_{\mathrm{atm1}}\right) \tag{28}$$

V_{cg0} – the initial volume of the cylinder, when the piston is in its front position,

S_c – the effective area of the piston cross section,

x_{pt} – the displacement of the piston in gas cylinder and parts linked with it.

i. <u>The equation of motion for piston and parts linked with it</u>

$$\frac{\mathrm{d}x_{\mathrm{pt}}}{\mathrm{d}t} = v_{\mathrm{pt}} \tag{29}$$

$$\frac{\mathrm{d}v_{\mathrm{pt}}}{\mathrm{d}t} = \frac{1}{M}\left[S_{\mathrm{c}}(p_{\mathrm{cg}} - p_{\mathrm{atm}}) - F_{\mathrm{sp}} - F_{\mathrm{f}}\right] \tag{30}$$

where

v_{pt} – the velocity of piston and other parts linked with it,

F_{sp} – the force of return spring.

$$F_{\mathrm{sp}} = F_{\mathrm{sp0}} + c_{\mathrm{sp}}x_{\mathrm{pt}} \tag{31}$$

F_{sp0} – the initial pre-stress of return spring,

c_{sp} – the return spring constant,

M – the mass of piston and other parts linked with it.

$$M = m_{\text{pt}} + m_{\text{bk}} + \frac{m_{\text{sp}}}{3} \tag{32}$$

m_{pt} – the mass of moveable drive piston,
m_{bk} – the mass of breech block carrier,
m_{sp} – the mass of return spring,
F_{f} – the friction force effect to the moving parts, see [9].

j. Energy balance equation in the gas cylinder

The energy equation in the gas cylinder is based on the first law of thermodynamics [10]:

$$\mathrm{d}Q_{\text{c}} = \mathrm{d}U_{\text{c}} + \sum \mathrm{d}L = \mathrm{d}U_{\text{c}} + p_{\text{cg}}\,\mathrm{d}V_{\text{cg}} \tag{33}$$

where

$\mathrm{d}Q_{\text{c}}/\mathrm{d}t$ – the enthalpy rates crossing the boundary in the gas cylinder

$$\mathrm{d}Q_{\text{c}} = \dot{H}_{\text{c}}\mathrm{d}t = \left(\dot{m}_{\text{c}}\mathrm{d}t\right)c_{\text{p}}T = \dot{m}_{\text{cg}}c_{\text{p}}T_{\text{in}}\mathrm{d}t - \dot{m}_{\text{atm1}}c_{\text{p}}T_{\text{cg}}\mathrm{d}t \tag{34}$$

$$\dot{m}_{\text{cg}}T_{\text{in}} = \begin{cases} 0 & p = p_{\text{cg}} \quad \text{or} \quad l \le l_{\text{p}} \\ \dot{m}_{\text{cg}}T & p > p_{\text{cg}} \quad \text{and} \quad l > l_{\text{p}} \\ \dot{m}_{\text{cg}}T_{\text{cg}} & p_{\text{cg}} > p \quad \text{and} \quad l > l_{\text{p}} \end{cases} \tag{35}$$

$\mathrm{d}U_{\text{c}}/\mathrm{d}t$ – the internal energy time change in the gas cylinder

$$\mathrm{d}U_{\text{c}} = \mathrm{d}(m_{\text{c}}c_{\text{v}}T_{\text{cg}}) = c_{\text{v}}\left(\dot{m}_{\text{c}}\mathrm{d}tT_{\text{cg}} + m_{\text{c}}\dot{T}_{\text{cg}}\mathrm{d}t\right) = c_{\text{v}}\left(\dot{m}_{\text{c}}T_{\text{cg}} + m_{\text{c}}\dot{T}_{\text{cg}}\right)\mathrm{d}t \tag{36}$$

By introducing Eqs (34)-(36) into Eq. (33), it yields

$$\frac{\mathrm{d}T_{\text{cg}}}{\mathrm{d}t} = \frac{1}{m_{\text{c}}}\left[\dot{m}_{\text{cg}}\left(\kappa T_{\text{in}} - T_{\text{cg}}\right) - T_{\text{cg}}\left(\kappa - 1\right)\dot{m}_{\text{atm1}} - \frac{p_{\text{cg}}}{c_{\text{v}}}S_{\text{c}}v_{\text{pt}}\right] \tag{37}$$

Finally, we summarize the system of differential equations and algebraic equations of the internal ballistics and drive of automatic mechanism of gas-operated machine gun: Eqs (1)-(12), (20), (24), (26), (29), (30), and (37). Initial conditions of this system of equations discussed above at the time is:
$t_0 = 0$; $v = 0$; $l = 0$; $T = T_{\text{atm}}$; $v_{\text{pt}} = 0$; $x_{\text{pt}} = 0$; $\psi = \psi_0$; $z = z_0$; $V = V_0$; $V_{\text{cg}} = V_{\text{cg0}}$; $F_{\text{sp}} = F_{\text{sp0}}$; $T_{\text{cg}} = T_{\text{atm}}$; $m_{\text{atm1}} = 0$; $m_{\text{atm2}} = 0$. T_{atm} is atmospheric temperature.

The system of differential equations of the model can be fully solved by numerical method with fourth-order Runge-Kutta method.

3 Application of Presented Equations on the 7.62 mm Machine Gun UK-59 and Ammunition 7.62×54 R.

The mathematical model presented above was applied to the UK-59 machine gun, which is shown in Fig. 3. The experimental results and the calculated results were compared to test the accuracy of the thermodynamic mathematical model.

The input data and initial parameters: The numerical values and the input parameters of the calculations were obtained by measurements and by estimation on the real 7.62 mm machine gun UK-59 and ammunition 7.62×54 R. However, the numbers of inputs are large so only the most important ones are mentioned in Tab. 1 [9]. The typical results of the thermodynamic mathematical model are presented in Figs 4-6.

Tab. 1 Input data for solving problem of machine gun UK-59

Parameter	**Value**
The sectional area of barrel [m^2]	47.3×10^{-6}
The gun caliber [m]	7.62×10^{-3}
The barrel length [m]	0.609
The propellant charge mass [kg]	3.10×10^{-3}
The projectile mass [kg]	9.6×10^{-3}
The propellant force [J kg^{-1}]	0.73×10^{6}
The propellant covolume [m^3 kg^{-1}]	0.906×10^{-3}
The propellant density [kg m^{-3}]	1627
The total impulse of the gas pressure [Pa s]	1.7020×10^{5}
The propellant ignition temperature [K]	3175
The initial volume of the barrel [m^3]	3.521×10^{-6}
The Poisson constant [–]	1.2505
The projectile starting pressure [Pa]	40×10^{6}
The shape characteristic quantity of fast burning propellant [–]	$\kappa_c = 1.092$ $\kappa_c\mu = -0.092$ $\lambda = 0$
The mass of ammunition [kg]	0.0189
The diameter of piston [m]	13.937×10^{-3}
The diameter of cylinder [m]	14.015×10^{-3}
The initial volume of cylinder [m^3]	12.363×10^{-7}
The diameter of gas port [m]	1.31×10^{-3}
The position of gas port [m]	0.18
The discharge coefficient φ_1 [–]	0.5572
The discharge coefficient φ_2 [–]	0.65
The discharge coefficient φ_3 [–]	0.1605
The discharge coefficient φ_5 [–]	0.98
The mass of breech [kg]	0.21952
The mass of breech block carrier [kg]	0.83028
The mass of recoil spring [kg]	0.068259
The initial pre-stress force [N]	61
The recoil spring constant [N m^{-1}]	666

Fig. 3 The machine gun UK-59

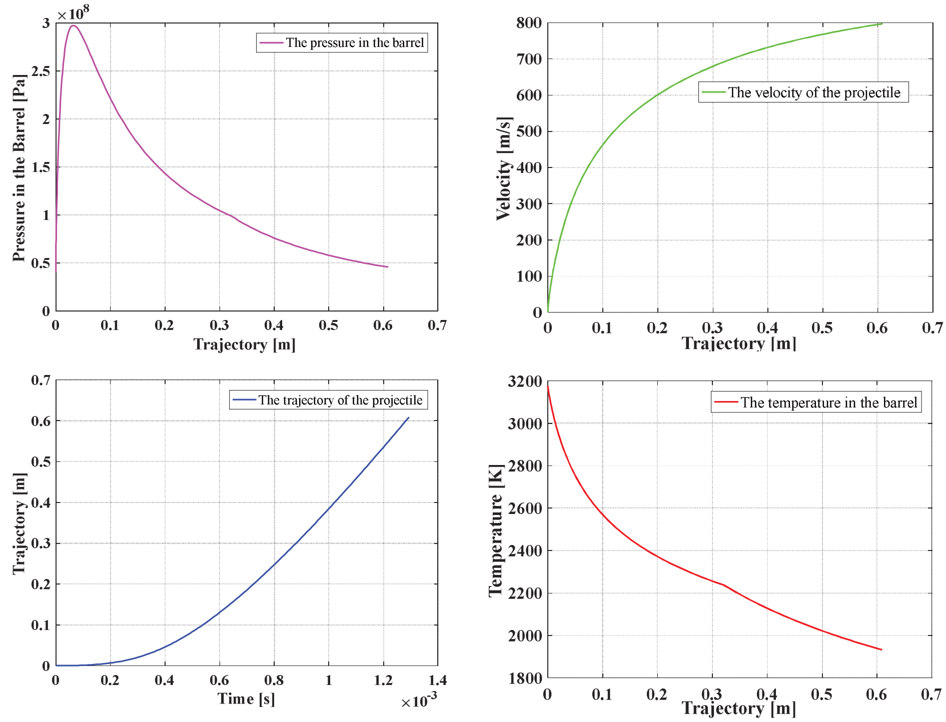

Fig. 4 Typical trajectory courses of pressure, temperature in the barrel and velocity of the projectile in the barrel, time courses of trajectory of the projectile in the barrel

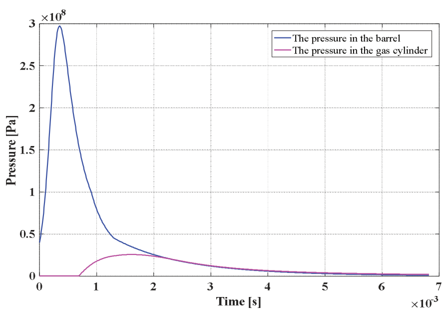

Fig. 5 Time courses of pressure

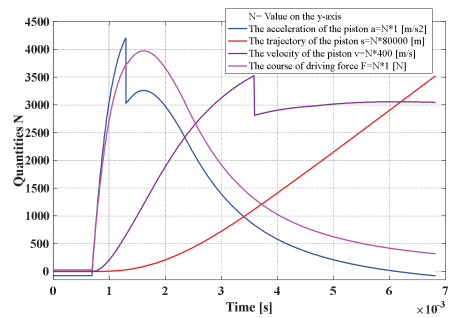

Fig. 6 Typical time courses of acceleration, trajectory, velocity of the piston and driving force

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4 Experiment

The experiment was performed at the laboratory shooting range of the Department of Weapons and Ammunition of the University of Defence (the Czech Republic). Experimental structure model and layout of measuring positions on the machine gun UK-59 is shown in Fig. 7 [9].

The pressure was measured by piezoelectric pressure sensors S1, S2, S3. The first piezoelectric pressure sensor S1 was mounted at the mouth of cartridge case to measure the gases pressure in the barrel. The second piezoelectric pressure sensor S2 was mounted above the gas ports. This sensor measured the gases pressure in the barrel at the gas ports. The third piezoelectric pressure sensor S3 was located on the front of the gas cylinder for measuring the gases pressure in the gas cylinder.

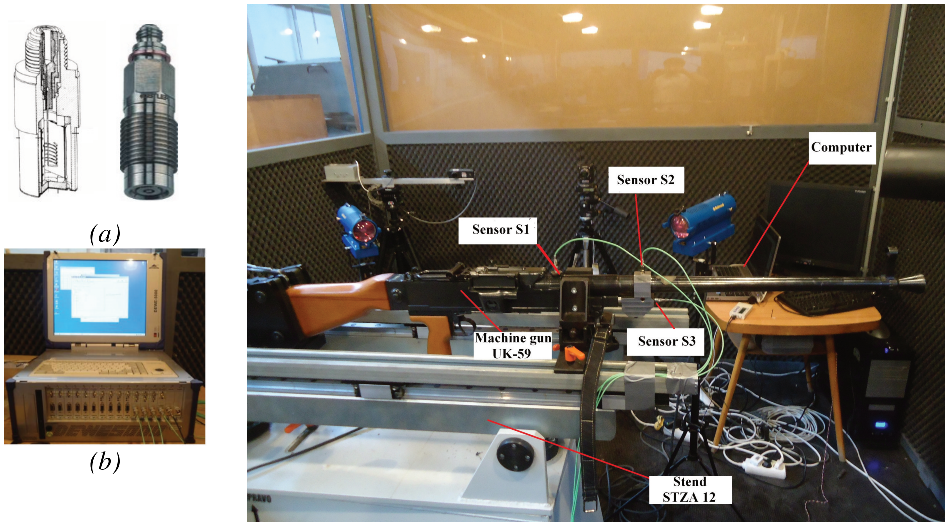

Fig. 7 (main figure) – 7.62 mm *machine gun UK-59 on STZA12, (a) – Piezoelectric sensor KISTLER, (b) – Measuring system DEWE – 500*

Carrying out the measurement and data processing, we have obtained the results of the experiment in Fig. 8.

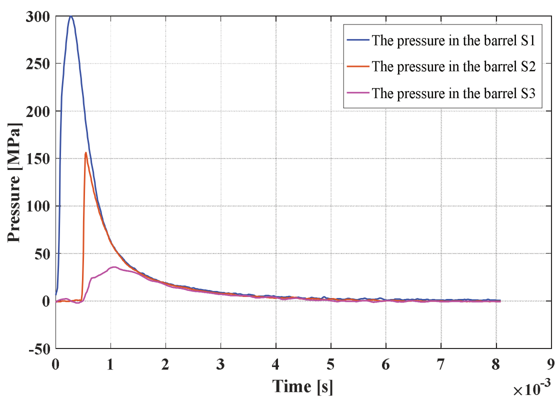

Fig. 8 Time histories of gas pressures in the barrel and in the gas cylinder

5 Discussion

The obtained experimental pressures were compared with the calculated pressure of gases propellant. The comparison results in the different diameters of gas ports are shown in Figs 9 and 10, and Tab. 2.

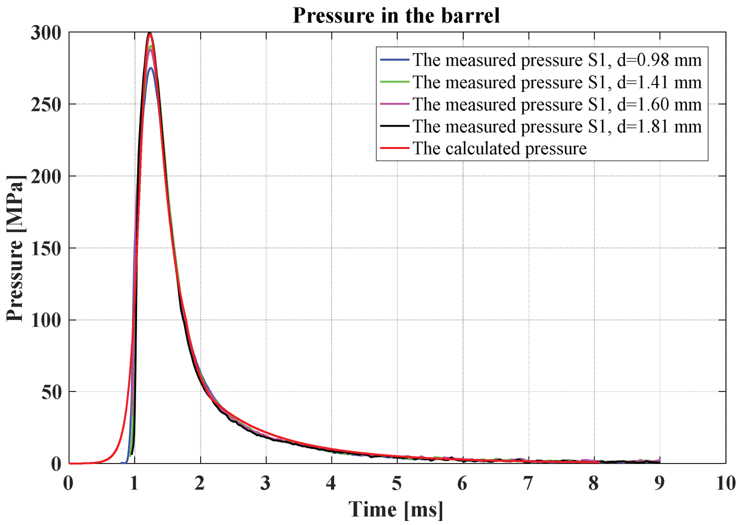

Fig. 9 Time course of gas pressure in the barrel

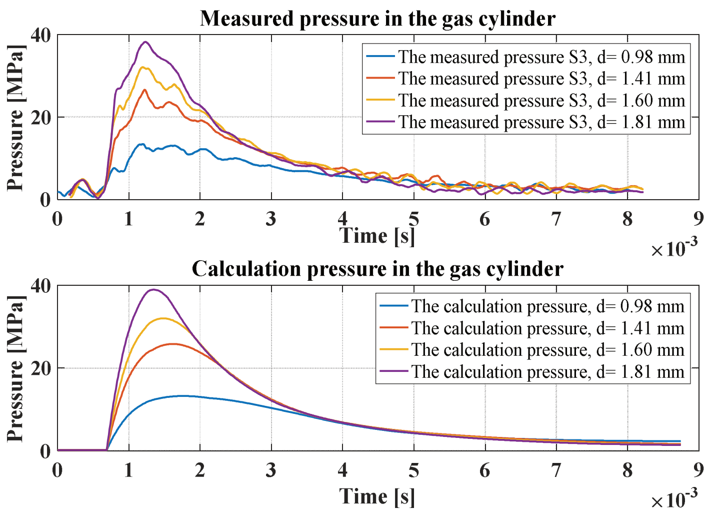

Fig. 10 Time course of gas pressure in the gas cylinder

The results of solving the mathematical thermodynamic model on the UK-59 machine gun are consistent with the measured results of the machine gun UK-59 in Figs 9 and 10, and Tab. 2

Tab. 2 Comparison of calculated and measured values with diameter of gas ports d = 1.81 mm (S_0 = 2.573 mm^2)

Parameter	**Calculated value** [MPa]	**Measured values** [MPa]	**Difference** [MPa]
Maximum pressure in the barrel	297.23	299.72	2.49
Maximum pressure in the gas cylinder	38.91	38.26	0.65
Pressure in the barrel at the gas ports	154.40	156.09	1.69

The research results confirm the correctness of the point of view of building physical and mathematical models, so this model can be applied in investigating the factors affecting the thermal-dynamic properties, internal ballistics, and gas drive machine of gas-operated machine guns.

6 Conclusion

In this paper, a thermodynamic mathematical model was developed to solve the internal ballistics and gas propulsion mechanism of automatic weapons. The novel model correctly and completely describes the internal ballistic cycle in the barrel and the phenomenon that occurs in the gas cylinder. The system of differential equations and algebraic equations has been built to solve general cases so that it can be applied to specific cases with similar structure. All results of the calculations in the example of 7.62 mm UK-59 machine gun and 7.62×54 R ammunition agree very well with the experimental results.

The results of the research confirm the correctness of the physical and thermodynamic mathematical models, so that they can be used to analyze the factors affecting the thermodynamic properties, internal ballistics and gas propulsion of these types of gas-operated automatic weapons.

The results of this paper are important for the calculation and design of an automatic weapon with gunpowder gas extraction and allow calculation according to the specific technical requirements for a particular automatic weapon.

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