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Theoretical and Technical Aspects of Optical Access Networks Construction

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Abstract:

There are numerous telecommunication operators in the world who have started to replace old metallic cable networks by networks based on cables with optical fibres or to build completely new optical networks which cost a lot of money. To provide guidance to these costs, general theoretical formulae have been derived on simplified assumptions which should enable the basic orientation to the costs that the construction of an optical access network requires. The theoretically obtained results are compared with the practical solution of a real optical access network.

Keywords

optical access network, optical fibre, splitter, optical distribution point, optical access node, radial, or rectangular network configuration, attraction area, subscriber

1. Introduction

When a telecommunication operator wants to provide quality, full valuable and faultless services with the best possible technical parameters, it has to build the access networks based on optical cables, too, like the core transmission networks had been built up at the end of the 20-th century connecting service and access nodes, and thus to get rid of the problems bearing upon crosstalk's and disturbances on metallic cables for ever. The optical access network enables to provide almost unlimited bandwidth to a subscriber, a practical range of tens kilometres, reliable and undisturbed information transmission, an extend network monitoring and an efficient network control. These things are an important part of the telecommunication network operator strategy.

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To achieve the above-mentioned goal, an optical fibre must be brought to each subscriber from a common distribution point of the optical access network. Optical fibres going from the distribution points farther up to an optical access node are shared by more subscribers (actually up to 128). There are two technologies known which operate on these principles:

- the actually used gigabit passive optical network technology [1],
- the future optical access network technology based on the optical wavelength division multiplex [2, 3].

Both up-to-date access technologies are able to create the future full optical telecommunication access network.

2. Basic Optical Access Network Structure

Let us consider a full optical access network with an optical access node which can achieve thousands of subscribers in the future. The subscriber's optical network terminations are connected by their own access optical fibres to the passive optical distribution points which contain one or more optical splitters. The splitters are then connected by means of shared optical fibres to the optical line terminations of an optical access node. While the access optical fibres between the optical network terminations and the splitters are only used by particular subscribers, the optical fibres between the splitters in the distribution points and the optical line termination are multi-used or shared by all subscribers connected to the splitter (Fig. 1).

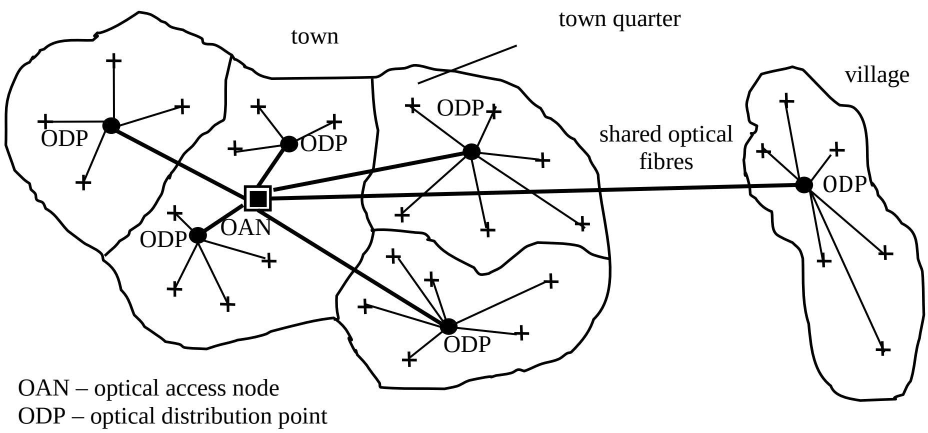

Fig. 1 Optical access network structure

One optical distribution point serves an area with a relatively high number of subscribers. It includes splitters and, additionally, it may distribute the shared optical fibres to other points of the access network (this type of distribution will not be further considered for simplicity) as it can be seen in Fig. 2.

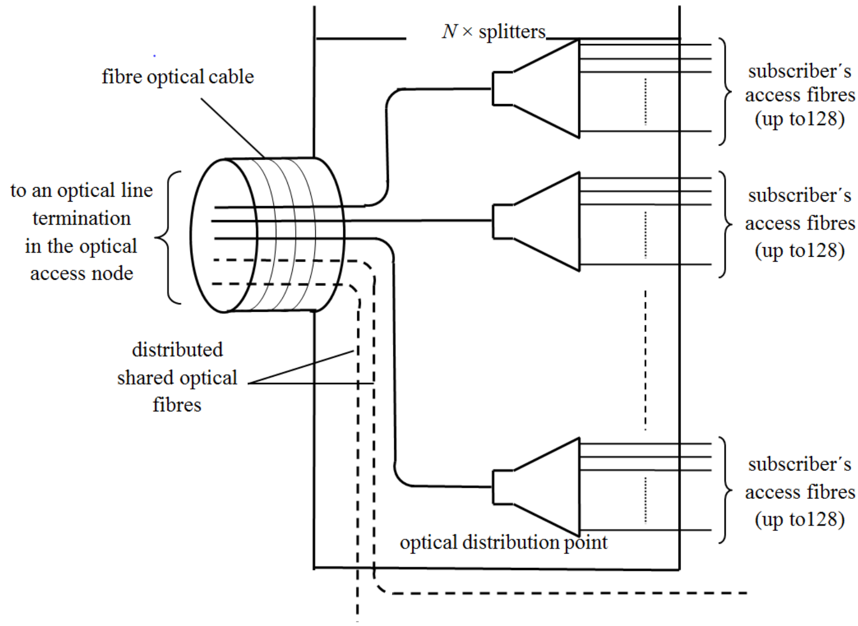

Fig. 2 Optical distribution point

3. Basic Presumptions

The first attempt to solve a similar problem applied on an obsolent simple metalic access network was made in [4].

To obtain explicit analytical results, the attraction areas covered by subscribers have to be approximated by a formation which can be mathematically described. The considered formations are:

- ellipse, circle or square as special types which are suitable for approximation of ward quarters or villages situated on a plane;
- rectangle which is suitable for approximation of straight streets or villages situated in valleys or an indoor optical cabling in high buildings with many stocks.

As to access optical fibres, two possible network configurations will be taken into consideration:

- the rectangular configuration which is suitable for rectangular crossing streets in a covered area or for an indoor optical cabling;
- the radial configuration which can approximate radial or irregular road and street arrangement.

4. List of Variables

The following variables will occur in the equations as they are listed below:
a – ellipse (rectangle) horizontal half axis
b – ellipse (rectangle) vertical half axis
r – circle radius

A, B, C, K, c, c_1, c_2 – integration coefficients and constants
h – centre shift
n – number of subscribers
N – number of shared fibres needed
p – split ratio
C_a – costs per length unit of an access fibre
C_s – costs per length unit of a shared (multi-used) fibre
C_T – total costs on access network construction in a given attraction area
x, y – Cartesian variables
t – parameter, auxiliary variable
ρ, φ – polar variables
Δ – Jacobian
l_x – entire length of access fibres along the x-axis
l_y – entire length of access fibres along the y-axis
l – total length of access fibres
l_s – entire length of shared (multi-used) optic fibres in an attraction area
S – surface of an attraction area:
The ellipse:

$$S = \pi ab \tag{1}$$

The circle

$$S = \pi r^2 \tag{2}$$

The rectangle:

$$S = 2a2b = 4ab \tag{3}$$

The square:

$$S = 4a^2 \tag{4}$$

The subscriber penetration (density) on condition of plane spread of subscribers:

$$s = \frac{n}{S} \tag{5}$$

5. General Formulae Derivations

5.1. Attraction Area Approximated by Ellipse (Circle) with Rectangle Access Network Configuration

As the first step, the entire length of access fibres along the x-axis l_x will be derived (Fig. 3). The ideal position for the optical distribution point would be in the centre of the attraction area (ellipse, circle), if shared fibres were not necessary. But the position of the optical distribution point is influenced by the number of the shared optical fibres necessary for interconnection of the optical distribution point and the optical access node.

The number of fibres that will be added from the elementary surface dS to other fibres going in the common optical cable duct will be the same as the elementary number of subscribers on the elementary surface dS:

$$\mathrm{d}n = s\mathrm{d}S = sy\mathrm{d}x \tag{6}$$

where y represents the analytical expression for the ellipse equation:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \tag{7}$$

$$y=\pm b\sqrt{1-\left(\frac{x}{a}\right)^2} \tag{8}$$

The farther the elementary section dx is situated from the distribution point O’, the higher the entire length of fibres will be. So the entire length of fibres in the elementary section dx is proportional to the distance of the elementary section from the distribution point O’. This entire elementary length linearly decreases when approaching from left to the distribution point O’ within $x \in \langle -a,\ h\rangle$ and increases when going out of the distribution point O’ within $x \in \langle h,\ a\rangle$ with the proportionality “constant” dn which depends on x:

$$\mathrm{d}n = sy\mathrm{d}x = \frac{n}{\pi ab}b\sqrt{1-\left(\frac{x}{a}\right)^2}\,\mathrm{d}x = \frac{n}{\pi a^2}\sqrt{a^2-x^2}\mathrm{d}x \tag{9}$$

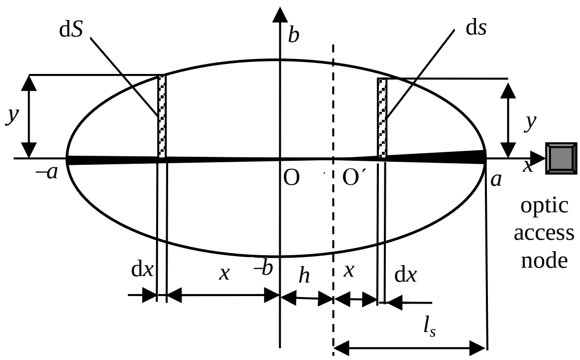

Fig. 3 Calculation of the entire length of access fibres along the x-axis

The linear decrease and increase of the entire lengths of fibres in the elementary sections dx on the left and on the right from the distribution point O´ respectively can be expressed as:

$$\mathrm{d}l_{x1} = \mathrm{d}n(-x+h) = \frac{n}{\pi a^2}(-x+h)\sqrt{a^2-x^2}\mathrm{d}x \tag{10}$$

$$\mathrm{d}l_{x2} = \mathrm{d}n(x-h) = \frac{n}{\pi a^2}(x-h)\sqrt{a^2-x^2}\mathrm{d}x \tag{11}$$

The entire optical fibre length in the common optical cable duct along the x-axis from the left and from the right of the distribution point O’ will be obtained by integrations:

$$l_x = l_{x1}+l_{x2} = 2\int_{-a}^{h}\mathrm{d}l_{x1} + 2\int_{h}^{a}\mathrm{d}l_{x2} =$$

$$= \frac{2n}{\pi a^2}\int_{0}^{a+h}\frac{-x^3+2hx^2+(a^2-h^2)x}{\sqrt{-x^2+2hx+a^2-h^2}}\mathrm{d}x + \frac{2n}{\pi a^2}\int_{0}^{a-h}\frac{-x^3-2hx^2+(a^2-h^2)x}{\sqrt{-x^2-2hx+a^2-h^2}}\mathrm{d}x \tag{12}$$

General solution of the integrals in equation (12) is [5]:

$$\int \frac{-x^3 \pm 2hx^2 + (a^2 - h^2)x}{\sqrt{-x^2 \pm 2hx + a^2 - h^2}}\,\mathrm{d}x =$$

$$= \left(\frac{x^2}{3} \mp \frac{h}{6} - \frac{a^2}{3} - \frac{h^2}{6}\right)\sqrt{a^2 - (\mp x + h)^2} - \frac{1}{2}a^2 h \arcsin\frac{\mp x + h}{a} + c \qquad (13)$$

And then

$$l_{x1} = \frac{2n}{\pi a^2}\left[\left(\frac{a^2}{3} + \frac{h^2}{6}\right)\sqrt{a^2 - h^2} + \frac{1}{2}a^2 h \arcsin\frac{h}{a} + \frac{\pi}{4}a^2 h\right] \qquad (14)$$

$$l_{x2} = \frac{2n}{\pi a^2}\left[\left(\frac{a^2}{3} + \frac{h^2}{6}\right)\sqrt{a^2 - h^2} + \frac{1}{2}a^2 h \arcsin\frac{h}{a} - \frac{\pi}{4}a^2 h\right] \qquad (15)$$

The entire optical fibre length in the common optical duct along the whole *x*-axis ($x \in \langle -a, a\rangle$) in the attraction area is:

$$l_x = \frac{2n}{\pi a^2}\left[2\left(\frac{a^2}{3} + \frac{h^2}{6}\right)\sqrt{a^2 - h^2} + \frac{1}{2}a^2 h \arcsin\frac{h}{a}\right] \qquad (16)$$

To find the optimal shift h of the distribution point towards the optical access node along the *x*-axis, the next condition must be fulfilled:

$$C_a l_{x1} = C_a l_{x2} + C_s l_s \qquad (17)$$

$$C_a (l_{x1} - l_{x2}) = C_s N(a - h) \qquad (18)$$

$$\left[\left(\frac{a^2}{3} + \frac{h^2}{6}\right)\sqrt{a^2 - h^2} + \frac{1}{2}a^2 h \arcsin\frac{h}{a} - \frac{\pi}{4}a^2 h\right] = C_s N(a - h) \qquad (19)$$

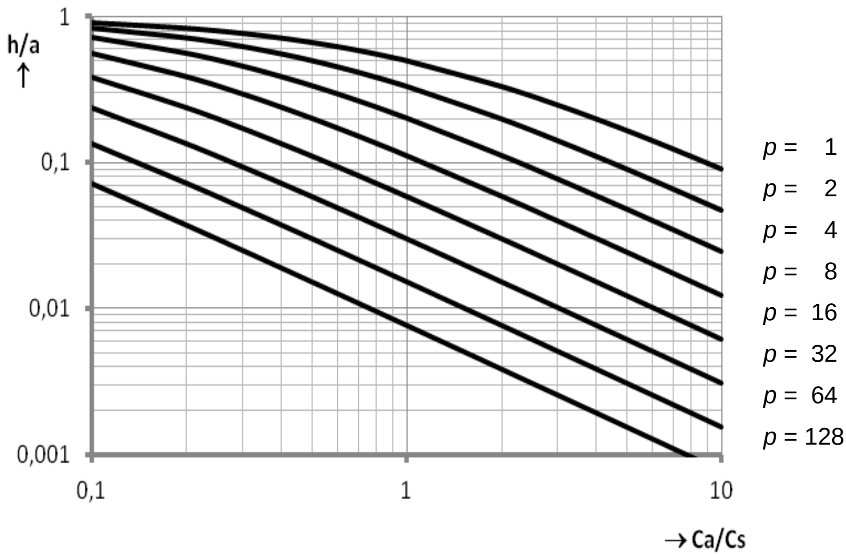

Fig. 4 Relative shift of the distribution point versus relative costs with the split ratio p as a parameter

And from there

$$\frac{C_a}{C_s} = \frac{N(a-h)}{nh} \tag{20}$$

$$\frac{h}{a} = \frac{1}{1 + \frac{n}{N}\frac{C_a}{C_s}} \tag{21}$$

This formula is plotted in Fig. 4 for the split ratio p = 1, 2, 4, 8, 16, 32, 64, 128.
As the ratio $n/N = p$ performs the optical split ratio, then

$$\frac{h}{a} = \frac{1}{1 + p\frac{C_a}{C_s}} \tag{22}$$

Assuming the highest split ratio p = 128 and the same costs on access and shared optical fibres ($C_a = C_s$), the relative shift

$$\frac{h}{a} = \frac{1}{1+128} = 0.007\ 75 \tag{23}$$

which is only approximately 0.8 % (or 8 m on the length of 1 000 m). Therefore the shift of the distribution point towards the optical access node will be neglected in further theoretical considerations.
Assuming $h = 0$, the entire length of optical fibres along the x-axis

$$l_x = \frac{4}{3\pi}na \tag{24}$$

As the second step, the entire length of access fibres along the y-axis will be derived on the assumption that the distribution point is placed in the centre of the attraction area (the shift h is supposed to be zero). The further assumption is that the optical fibres fall into the common optical duct along the x-axis vertically and so the elementary fibre length l_y from the elementary surface dS will be (Fig. 5):

$$\mathrm{d}l_y = y\mathrm{d}n = ys\mathrm{d}S = \frac{n}{\pi ab}y\mathrm{d}x\mathrm{d}y \tag{25}$$

$$l_y = 4\frac{n}{\pi ab}\int_0^a \int_0^{b\sqrt{1-\left(\frac{x}{a}\right)^2}} y\mathrm{d}x\mathrm{d}y = \frac{4}{3\pi}nb \tag{26}$$

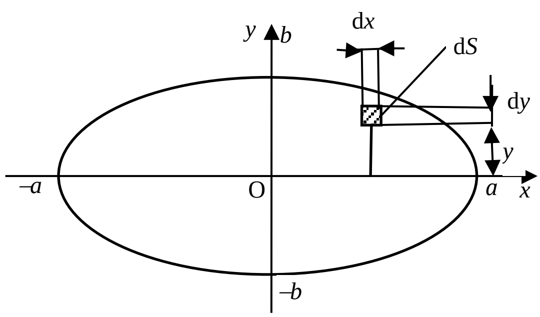

Fig. 5 Calculation of the entire length of access fibres along the y-axis

And the total length of the access network:

$$l = l_x + l_y = \frac{4}{3\pi}(a+b) = \frac{4}{3\pi} na\left(1+\frac{b}{a}\right) \tag{27}$$

The total costs for the considered network configuration:

$$C_T = \left[\frac{4}{3\pi}\left(1+\frac{b}{a}\right)C_a + \frac{C_s}{p}\right]na \tag{28}$$

For the special case of a circular area (a = b = r):

$$l = \frac{8}{3\pi} nr \tag{29}$$

$$C_T = \left(\frac{8}{3\pi}C_a + \frac{C_s}{p}\right)nr \tag{30}$$

Costs variations:

$$C_T = \left[\left(\frac{4}{3\pi} \div \frac{8}{3\pi}\right)C_a + \frac{C_s}{p}\right]na \tag{31}$$

5.2. Attraction Area Approximated by Rectangle (Square) with Rectangular Access Network Configuration

The calculation procedure is similar to that in Chapter 5.1, only attraction area is simpler (Fig. 6):

$$l_{x1} = 2\frac{n}{2a \cdot 2b}\int_{-a}^{h}(-x+h)b\mathrm{d}x = \frac{n}{2a}\left(\frac{a^2}{2}+\frac{h^2}{2}+ah\right) \tag{32}$$

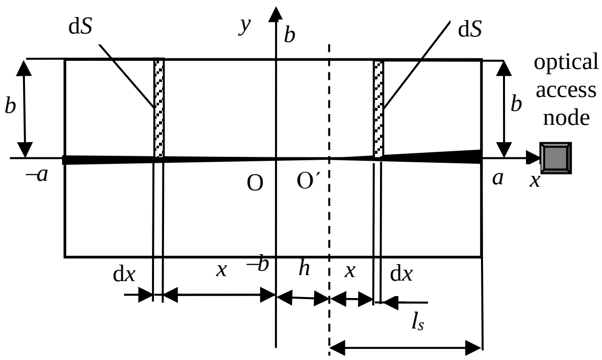

Fig. 6 Calculation of the total length of optical fibres in the access network with the rectangle (square) shape with rectangular configuration

$$l_{x2} = 2\frac{n}{2a \cdot 2b}\int_{h}^{a}(x-h)b\mathrm{d}x = \frac{n}{2a}\left(\frac{a^2}{2}+\frac{h^2}{2}-ah\right) \tag{33}$$

$$l_x = l_{x1} + l_{x2} = \frac{n}{2a}(a^2+h^2) \tag{34}$$

The possible optimal shift of the distribution point is the same as in the elliptic attraction area, as it is evident from the next equations:

$$C_a(l_{x1} - l_{x2}) = C_s \frac{n}{p}(a - h) \tag{35}$$

$$\frac{h}{a} = \frac{1}{1 + p\frac{C_a}{C_s}} \tag{36}$$

When $h = 0$, then

$$l_x = \frac{1}{2}na \tag{37}$$

and

$$l_y = 4s\int_0^a\int_0^b y\mathrm{d}x\mathrm{d}y = \frac{4n}{4ab}\int_0^a\left(\int_0^b y\mathrm{d}y\right)\mathrm{d}x = \frac{1}{2}nb \tag{38}$$

$$l = l_x + l_y = \frac{1}{2}n(a+b) = \frac{1}{2}na\left(1 + \frac{b}{a}\right) \tag{39}$$

$$C_T = \left[\frac{1}{2}\left(1 + \frac{b}{a}\right)C_a + \frac{C_s}{p}\right]na \tag{40}$$

For the special case of the square area ($a = b$):

$$l = na \tag{41}$$

$$C_T = \left(C_a + \frac{C_s}{p}\right)na \tag{42}$$

Costs variations:

$$C_T = \left[\left(\frac{1}{2} \div 1\right)C_a + \frac{C_s}{p}\right]na \tag{43}$$

5.3. Attraction Area Approximated by Ellipse (Circle) with Rectangular Access Network Configuration

The entire length of access fibres along the radius will be derived on the assumption that the distribution point is located in the centre of the attraction area (Fig. 7).

The number of fibres outgoing from an elementary surface dS depends on the elementary number of subscribers within this surface:

$$\mathrm{d}n = s\mathrm{d}S = \frac{n}{\pi ab}\mathrm{d}x\mathrm{d}y \tag{44}$$

The entire elementary length dl of fibres outgoing from the elementary surface dS is proportional to the distance ρ of the elementary surface dS from the centre of the attraction area where the distribution point is located:

$$\mathrm{d}l = \rho\mathrm{d}n = \frac{n}{\pi ab}\rho\mathrm{d}x\mathrm{d}y = \frac{n}{\pi ab}\sqrt{x^2 + y^2}\mathrm{d}x\mathrm{d}y \tag{45}$$

The total length of optical fibres in the access network of this configuration will be obtained by integration:

$$l = 4\frac{n}{\pi ab}\int_0^a \int_0^{b\sqrt{1-\left(\frac{x}{a}\right)^2}} \sqrt{x^2+y^2}\,\mathrm{d}x\mathrm{d}y \tag{46}$$

Change from the Cartesian to the polar co-ordinates shall be made [5]:

$$\frac{x}{a} = \rho\cos\varphi \tag{47}$$

$$\frac{y}{b} = \rho\sin\varphi \tag{48}$$

Jacobi determinant of transformation:

$$\Delta = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} a\cos\varphi & -a\rho\sin\varphi \\ b\sin\varphi & b\rho\cos\varphi \end{vmatrix} = ab\rho \tag{49}$$

$$\rho \in \langle 0, 1\rangle$$

$$\varphi \in \langle 0, \frac{\pi}{2}\rangle$$

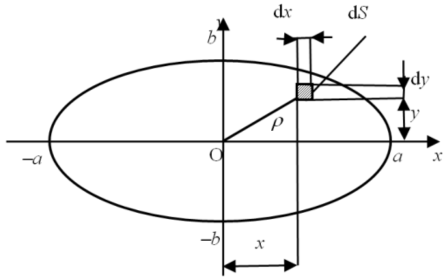

Fig. 7 Calculation of the total length of optical fibres in the access network with the ellipse (circle) shape with radial configuration

After substitution

$$l = \frac{4n}{\pi ab}\int_0^1 \int_0^{\frac{\pi}{2}} \sqrt{a^2\rho^2\cos^2\varphi + b^2\rho^2\sin^2\varphi}\,\Delta\mathrm{d}\varphi\mathrm{d}\rho =$$

$$= \frac{4}{\pi}na\int_0^1 \rho^2\mathrm{d}\rho \int_0^{\frac{\pi}{2}} \sqrt{\cos^2\varphi + \left(\frac{b}{a}\right)^2\sin^2\varphi\mathrm{d}\varphi} = \frac{4}{3\pi}na\int_0^{\frac{\pi}{2}} \sqrt{\cos^2\varphi + \left(\frac{b}{a}\right)^2\sin^2\varphi}\,\mathrm{d}\varphi \tag{50}$$

The integral (50) can only be solved numerically. Its numeric value depends on the b/a ratio. This dependency can be found in Fig. 10 (Curve 3).

In case of $b = 0$

$$l = \frac{4}{3\pi}na\int_0^{\frac{\pi}{2}} \cos\varphi\mathrm{d}\varphi = \frac{4}{3\pi}na \tag{51}$$

In case of $b = a = r$, the ellipse changes to the circle and then

$$l = \frac{4}{3\pi} nr \int_0^{\frac{\pi}{2}} \mathrm{d}\varphi = \frac{2}{3} nr \tag{52}$$

Curve 3 in Fig. 10 can be approximated by the straight line and then

$$l \approx (0.424a + 0.242b)n \tag{53}$$

The total costs and costs variation for the considered network configuration are:

$$C_T = \left[\left(\frac{4}{3\pi} \div \frac{2}{3}\right) C_a + \frac{C_s}{p}\right] na \tag{54}$$

5.4. *Attraction Area Approximated by the Rectangle (Square) With Radial Access Network Configuration*

The task to be solved is the same under the same conditions as in the previous chapter, only the shape of the attraction area is different (Fig. 8).

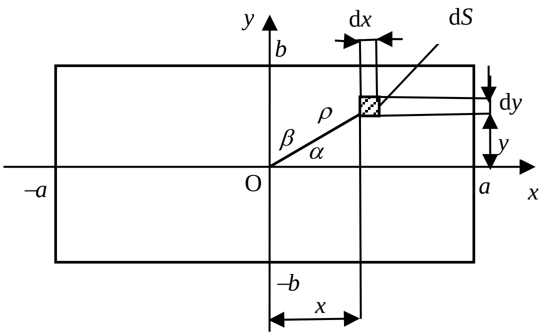

Fig. 8 Calculation of the total length of optical fibres in the access network with the rectangle (square) shape with radial configuration

The computation procedure is similar to that in the previous chapter:

$$\mathrm{d}l = \rho \mathrm{d}n = \rho s \mathrm{d}S = \rho \frac{n}{2a2b} \mathrm{d}x\mathrm{d}y = \frac{n}{4ab} \sqrt{x^2 + y^2} \mathrm{d}x\mathrm{d}y \tag{55}$$

$$l = 4\frac{n}{4ab} \int_0^a \int_0^{\frac{b}{a}x} \sqrt{x^2 + y^2} \mathrm{d}x\mathrm{d}y + \frac{4n}{4ab} \int_0^{\frac{a}{b}y} \int_0^b \sqrt{x^2 + y^2} \mathrm{d}x\mathrm{d}y \tag{56}$$

Introduction of the polar co-ordinates:

$$x = \rho \, cos \, \varphi \tag{57}$$

$$y = \rho \, sin \, \varphi \tag{58}$$

Transformation Jacobian:

$$\Delta = \begin{vmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial \rho} & \frac{\partial y}{\partial \varphi} \end{vmatrix} = \begin{vmatrix} \cos\varphi & -\rho\sin\varphi \\ \sin\varphi & \rho\cos\varphi \end{vmatrix} = \rho \tag{59}$$

where

$$\rho \in \langle 0, \frac{a}{\cos\varphi} \rangle$$

$$\varphi \in \langle 0, \arctan\frac{b}{a} \rangle$$

in the 1st integral and

$$\rho \in \langle 0, \frac{b}{\cos\varphi} \rangle$$

$$\varphi \in \langle 0, \arctan\frac{a}{b} \rangle$$

in the 2nd integral. Then:

$$l = \frac{n}{ab} \int_0^{\arctan\frac{b}{a}} \int_0^{\frac{a}{\cos\varphi}} \rho^2 \mathrm{d}\rho \, \mathrm{d}\varphi + \frac{n}{ab} \int_0^{\arctan\frac{a}{b}} \int_0^{\frac{b}{\cos\varphi}} \rho^2 \mathrm{d}\rho \, \mathrm{d}\varphi =$$

$$= \frac{1}{3} . \frac{n}{ab} \left(a^3 \int_0^{\arctan\frac{b}{a}} \frac{\mathrm{d}\varphi}{\cos^3\varphi} + b^3 \int_0^{\arctan\frac{a}{b}} \frac{\mathrm{d}\varphi}{\cos^3\varphi} \right) \tag{60}$$

The explicit solution of integrals in brackets is obtained using the per-partes method:

$$u = \frac{1}{\cos\varphi} \tag{61}$$

$$v' = \frac{1}{\cos^2\varphi} \tag{62}$$

in the first step and using the substitution

$$\sin\varphi = t \tag{63}$$

in the second step. And so:

$$\int \frac{\mathrm{d}\varphi}{\cos^3\varphi} = \frac{\sin\varphi}{2\cos^2\varphi} - \frac{1}{4}\ln\frac{1-\sin\varphi}{1+\sin\varphi} + c \tag{64}$$

Let us mark (see Fig. 8):

$$\arctan\frac{b}{a} = \alpha \tag{65}$$

$$\arctan\frac{a}{b} = \beta = \frac{\pi}{2} - \alpha \tag{66}$$

Then the total length of optical fibres will be:

$$l = \frac{1}{3}\frac{n}{ab}\left(a^3 \int_0^{\alpha} \frac{\mathrm{d}\varphi}{\cos^3\varphi} + b^3 \int_0^{\frac{\pi}{2}-\alpha} \frac{\mathrm{d}\varphi}{\cos^3\varphi} \right) = \frac{1}{3} na \frac{1}{\frac{b}{a}} \left[\frac{\sin\alpha}{2\cos^2\alpha} - \frac{1}{4}\ln\frac{1-\sin\alpha}{1+\sin\alpha} + \right.$$

$$\left. + \left(\frac{b}{a}\right)^3 \left(\frac{\cos\alpha}{2\sin^2\alpha} - \frac{1}{4}\ln\frac{1-\cos\alpha}{1+\cos\alpha} \right) \right] \tag{67}$$

In case of the square, $a = b$, $\alpha = \pi/4$ and:

$$l = \frac{1}{3}\left(\sqrt{2} - \frac{1}{2}\ln\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)na \tag{68}$$

When $b = 0$, $b/a = 0$, $\alpha = 0$

$$l = \frac{1}{2}na \tag{69}$$

The dependency $l/n/a$ on b/a from equation (67) is plotted in Fig. 10.

The total costs for the considered network configuration vary as:

$$C_T = \left\{\left[\frac{1}{2} \div \frac{1}{3}\left(\sqrt{2} - \frac{1}{2}\ln\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right]C_a + \frac{C_s}{p}\right\}na \tag{70}$$

5.5. Circle Attraction Area with Decreasing Penetration and with Radial Network Configuration

In this case, it is supposed that the subscriber penetration is not the same in the whole area any more. It decreases towards the area edge. Nor the penetration decrease (penetration change) is the same in any distance from the centre of the area. In other words, it is not linear; on the contrary, the penetration decrease depends on the value of the penetration that it has in a radial distance r from the centre which can be written as follows:

$$s(\rho) = -r\frac{\mathrm{d}s(\rho)}{\mathrm{d}\rho} \tag{71}$$

The proportion constant r is the radius of the area because the larger the covered area is, the faster the decrease (penetration change) must be. So we have got the linear differential equation with the constant coefficient r. Its solution is:

$$\mathrm{d}\rho = -r\frac{\mathrm{d}s(\rho)}{s(\rho)} \tag{72}$$

$$\rho = -r\ln\left|\frac{s(\rho)}{c}\right| \tag{73}$$

$$s(\rho) = c\mathrm{e}^{-\frac{\rho}{r}} \tag{74}$$

For the area centre, $\rho = 0$ and

$$s(0) = \frac{n}{\pi r^2} \tag{75}$$

$$\frac{n}{\pi r^2} = c \tag{76}$$

$$s(\rho) = \frac{n}{\pi r^2}\mathrm{e}^{-\frac{\rho}{r}} \tag{77}$$

The solution for the total access network length is as follows (Fig. 9):

$$\mathrm{d}l = \rho\mathrm{d}n = \rho s(\rho)\mathrm{d}S = \rho\frac{n}{\pi r^2}\mathrm{e}^{-\frac{\rho}{r}}\rho\mathrm{d}\rho\mathrm{d}\varphi \tag{78}$$

$$l = \frac{4n}{\pi r^2}\int_0^{\frac{\pi}{2}}\int_0^r \rho^2\mathrm{e}^{-\frac{\rho}{r}}\mathrm{d}\rho\mathrm{d}\varphi = \frac{2n}{r^2}\left[-\frac{r^3}{\mathrm{e}} - \frac{2r^3}{\mathrm{e}} + 2r^2\left(-\frac{r}{\mathrm{e}} + r\right)\right] = 2\left(2 - \frac{5}{\mathrm{e}}\right)nr \tag{79}$$

Having compared the total access network lengths in the previous chapters, it is obvious that the total access network length under the circumstances in this chapter is the lowest one from all. And the total costs will be lowest, too:

$$C_T = \left[2\left(2-\frac{5}{\mathrm{e}}\right)C_a + \frac{C_s}{p}\right]nr \tag{80}$$

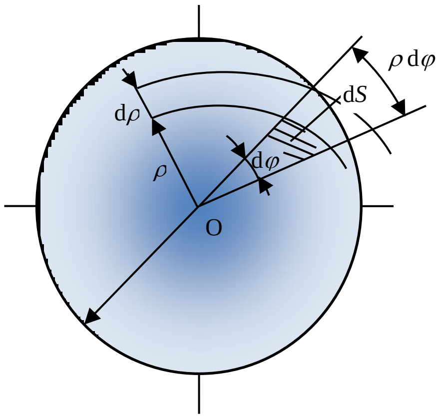

Fig. 9 Calculation of the total access network length in a circle area

6. Summary

The achieved results are summarized in Fig. 10 and in Table 1. Fig. 10 shows how the total access network length related to one subscriber depends on the size of the covered area. Table 1 states the numeric constants occurring in the formulae derived in the particular chapters that enable to estimate numerically the total length of the optical access network at assumed shapes of attraction areas.

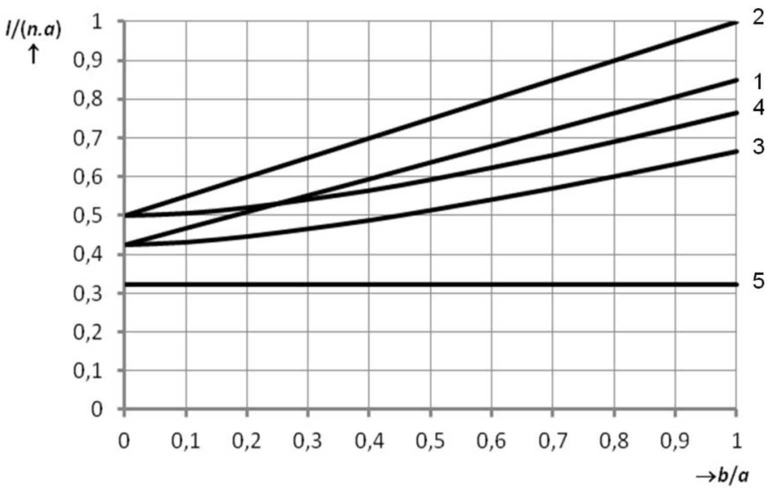

Fig. 10 Relative total lengths of optical access networks depending on their size, shape and configuration

Legend:

1 – ellipse (circle) shape with rectangular network configuration
2 – rectangle (square) shape with rectangular network configuration
3 – ellipse (circle) shape with radial network configuration
4 – rectangle (square) shape with radial network configuration
5 – circle shape with radial network configuration ($a = r$ – the radius of the circle area) and with non-linear subscriber penetration

As it can be seen from summarisation in Table 1, the most convenient would be the network with the circle shape with radial network configuration and with non-linear subscriber penetration, while the most inconvenient the network with the rectangle shape and with the rectangle rectangular configuration.

But the comparison of the network approximated by ellipse with rectangular configuration and the network approximated by rectangle with radial configuration is interesting. The total length of the access optical fibres in the first network is shorter than that of the latter one up to ratio b/a = 0.25. The situation gets inverse over this ratio, up to b/a =1.

Tab.1 Estimation constants

Chapter	**Area shape**	**Network configuration**	
		rectangle	**radial**
5.1, 5.3	ellipse	0.424 ÷ 0.849	0.424 ÷ 0.667
5.1, 5.3	circle	0.849	0.667
5.2, 5.4	rectangle	0.5 ÷ 1	0.500 ÷ 0.765
5.2, 5.4	square	1	0.765
5.5	circle	–	0.321

7. Comparison of Theoretical and Practical Solutions

To verify trustworthiness of the derived formulae in Chapter 5, the real attraction area in the town quarter Radvaň of Banská Bystrica was taken into account. The map of this area together with the designed access optical network is depicted in Fig. 11. As it is evident from Fig. 11, the rectangular network configuration and rectangle approximation of the attraction area could be the most convenient. Its dimensions are: length × width ($2a \times 2b$) = 264 × 100 m with the flats number, n = 370. Splitters 1:64 were used in the network design, so the split ratio p = 64 and costs $C_a = C_s = C$ were considered. The total length of the optical fibres was 32 661 m.

The total costs according to the formula (40) in Chapter 5.2. are:

$$C_T = C\left[\frac{1}{2}\left(1+\frac{b}{a}\right)+\frac{1}{p}\right]na = C\left[\frac{1}{2}\left(1+\frac{50}{132}\right)+\frac{1}{64}\right]370\cdot 132 = 34\,433C \qquad (81)$$

Relative error

$$e = \left|\frac{C(34\,433 - 32\,661)}{32\,661C}\right| = 5\ \% \qquad (82)$$

This comparison gives a good compliance of the theoretical and practical solutions.

Fig. 11 Map of solved attraction area

8. Conclusion

The future situation in telecommunications tends not only to unify technologies, but to unify network operators, too. And thus the situation may occur that a big network operator will operate in a large unified telecommunication network spread even over more countries for many telecommunication service providers. Such incumbent network operator will have to build up-to-date optical access networks for lots of customers. It may find the results derived in this paper useful for cost estimation purposes.

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References

[1] ČEPCIANSKY, G. and SCHWARTZ, L. *Technológia GPON*. EDIS – vydavateľstvo ŽU, Žilina, 2011. ISBN 978-80-554-0323-6.
[2] BAŠTIGÁL, M. *Teoreticko-technické aspekty budovania optických prístupových sietí*. [Diploma Thesis]. STU Bratislava, 2013, 57 p.
[3] RADOS, I., TURALIJA, P. and SUNARIC, T. WDM in SDH Network – Introduction and Interworking. *Proceedings of PHOTONICS 2002*. Prague, 2002. ISSN 0277-786X.
[4] TRSTENSKÝ, D. Ekonomický výpočet použitia koncentrátorov. *Sborník prací VŠD a VÚD* 24. Praha: NADAS, 1970.
[5] ŠKRÁŠEK, J. and TICHÝ, Z. *Základy aplikované matematiky I, II, III*. SNTL Praha, 1989. ISBN 80-03-00150-1